Series ABDC2/4

Set No. 2

प्रश्न-पत्र कोड
Q.P. Code 65/4/2

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ* ***7*** *हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में* ***14*** *प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains* ***7*** *printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains* ***14*** *questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे *अधिकतम अंक : 40*

Time allowed : 2 hours *Maximum Marks : 40*

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र के **तीन** खण्ड हैं – **खण्ड क, ख** तथा **ग** ।*

(ii) *प्रत्येक खण्ड **अनिवार्य** है ।*

(iii) ***खण्ड क** में **6** लघु उत्तर I प्रकार के प्रश्न हैं जिनमें प्रत्येक के **2** अंक हैं ।*

(iv) ***खण्ड ख** में **4** लघु उत्तर II प्रकार के प्रश्न हैं जिनमें प्रत्येक के **3** अंक हैं ।*

(v) ***खण्ड ग** में **4** दीर्घ-उत्तरीय प्रकार के प्रश्न हैं जिनमें प्रत्येक के **4** अंक हैं ।*

(vi) *कुछ प्रश्नों में आंतरिक विकल्प दिया गया है ।*

(vii) *प्रश्न सं. **14** एक प्रकरण-अध्ययन आधारित प्रश्न है जिसमें दो भाग हैं । प्रत्येक भाग के **2** अंक हैं ।*

खण्ड क

*प्रश्न संख्या **1** से **6** तक प्रत्येक प्रश्न के **2** अंक हैं ।*

1. एक समतल पर बिन्दु $(-2, -1, -3)$ से डाले गए लम्ब के पाद के निर्देशांक $(1, -3, 3)$ हैं । समतल का समीकरण ज्ञात कीजिए । *2*

2. एक सिक्का दो बार उछाला जाता है । निम्नलिखित तालिका में पटों की संख्या का प्रायिकता बंटन दिखाया गया है :

X	0	1	2
P(X)	K	6K	9K

(क) K का मान ज्ञात कीजिए ।

(ख) क्या यह सिक्का अभिनत या अनभिनत है ? अपने उत्तर का औचित्य दीजिए । *2*

3. (क) यदि $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ और $|\vec{b}| = 5$ है, तो $|\vec{a}|$ का मान ज्ञात कीजिए । *2*

अथवा

(ख) परिमाण $5\sqrt{3}$ वाले वे सभी संभव सदिश ज्ञात कीजिए जो निर्देशांक-अक्षों के साथ एकसमान कोण बनाते हैं । *2*

4. अवकल समीकरण $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$ का व्यापक हल ज्ञात कीजिए । *2*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Questions number **1** to **6** carry **2** marks each.*

1. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane. *2*

2. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer. *2*

3. (a) If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{b}| = 5$, then find the value of $|\vec{a}|$. *2*

OR

(b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes. *2*

4. Find the general solution of the differential equation

$$\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0.$$ *2*

5. मान ज्ञात कीजिए : *2*

$$\int_0^1 x^2 e^x \, dx$$

6. दो थैले दिए गए हैं । थैले I में 1 लाल और 3 सफेद गेंदें और थैले II में 3 लाल और 5 सफेद गेंदें हैं । एक थैले को यादृच्छया चुना जाता है और उसमें से एक गेंद निकाली जाती है । निकाली गई गेंद के लाल रंग की होने की प्रायिकता ज्ञात कीजिए । *2*

खण्ड ख

*प्रश्न संख्या **7** से **10** तक प्रत्येक प्रश्न के **3** अंक हैं ।*

7. समाकलन के प्रयोग से, क्षेत्र $\{(x, y) : y^2 \leq x \leq y\}$ का क्षेत्रफल ज्ञात कीजिए । *3*

8. (क) यदि एक रेखा x-अक्ष तथा z-अक्ष की धनात्मक दिशाओं से क्रमश: 60° तथा 45° के कोण बनाती है, तो वह कोण ज्ञात कीजिए जो यह रेखा y-अक्ष की धनात्मक दिशा से बनाती है । अत: रेखा की दिक्-कोसाइन लिखिए । *3*

अथवा

(ख) जाँच कीजिए कि क्या रेखाएँ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ और $\frac{x-4}{5} = \frac{y-1}{2} = z$ विषमतलीय हैं या नहीं । *3*

9. (क) ज्ञात कीजिए : *3*

$$\int \frac{1}{e^x + 1} \, dx$$

अथवा

(ख) मान ज्ञात कीजिए : *3*

$$\int_1^4 \{ |x| + |3 - x| \} \, dx$$

10. यदि $\vec{a}$ और $\vec{b}$ दो समान परिमाण के सदिश हैं और उनके बीच का कोण α है, तो सिद्ध कीजिए कि $\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = \cot\left(\frac{\alpha}{2}\right)$. *3*

5. Evaluate : *2*

$$\int_0^1 x^2 e^x \, dx$$

6. There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. Using integration, find the area of the region $\{(x, y) : y^2 \leq x \leq y\}$. *3*

8. (a) If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. *3*

OR

(b) Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not. *3*

9. (a) Find : *3*

$$\int \frac{1}{e^x + 1} \, dx$$

OR

(b) Evaluate : *3*

$$\int_1^4 \{ |x| + |3 - x| \} \, dx$$

10. If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = \cot\left(\frac{\alpha}{2}\right)$. *3*

खण्ड ग

*प्रश्न संख्या **11** से **14** तक प्रत्येक प्रश्न के **4** अंक हैं ।*

11. (क) अवकल समीकरण $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि $y(1) = 0$. *4*

अथवा

(ख) अवकल समीकरण $x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$ का व्यापक हल ज्ञात कीजिए । *4*

12. मान ज्ञात कीजिए : *4*

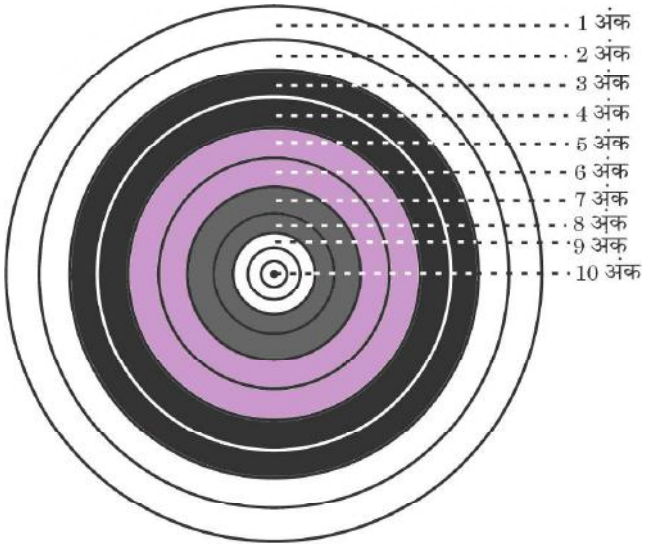
$$\int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}\,dx$$

13. समतलों $\vec{r}\cdot(\hat{i} + 3\hat{j}) = 6$ और $\vec{r}\cdot(3\hat{i} - \hat{j} - 4\hat{k}) = 0$ की प्रतिच्छेदन रेखा से गुज़रने वाले और मूल-बिन्दु से 1 इकाई की दूरी वाले समतलों के समीकरण ज्ञात कीजिए । *4*

प्रकरण-अध्ययन आधारित प्रश्न

14. तीरंदाज़ी के खेल में, तीरंदाज़ी लक्ष्य की प्रत्येक रिंग के लिए अंक निर्धारित किए गए हैं । केंद्रीय रिंग के 10 अंक और बाकी सभी रिंगों को 9 से 1 अंक, बाहर की ओर अनुक्रमिक क्रम में, दिए गए हैं ।

तीरंदाज़ A के 10 अंक प्राप्त करने की प्रायिकता 0·8 और तीरंदाज़ B के 10 अंक प्राप्त करने की प्रायिकता 0·9 है ।



उपरोक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

यदि दोनों तीरंदाज़, तीरंदाज़ी लक्ष्य को निशाना बनाते हैं, तो प्रायिकता ज्ञात कीजिए, जब

(क) केवल एक ही तीरंदाज़ 10 अंक प्राप्त करता है । *2*

(ख) दोनों तीरंदाज़ 10 अंक प्राप्त करते हैं । *2*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. (a) Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$, given that $y(1) = 0$. *4*

OR

(b) Find the general solution of the differential equation
$x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$. *4*

12. Evaluate : *4*

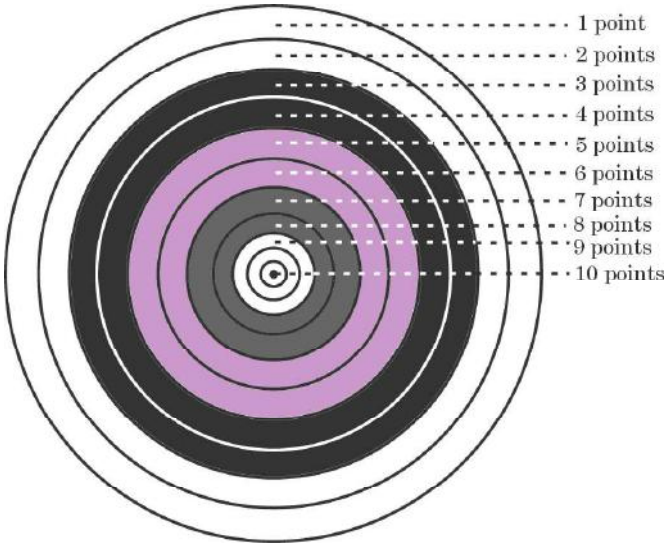
$$\int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}\,dx$$

13. Find the equations of the planes passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) = 6$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which are at a distance of 1 unit from the origin. *4*

Case-Study Based Question

14. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.



Based on the above information, answer the following questions :
If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points. *2*

(b) both of them earn 10 points. *2*

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Senior Secondary School Term II Examination, 2022

Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)

(PAPER CODE – 65/4/2)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks _____(example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-

    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totalling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totalling on the title page.
    - Wrong totalling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

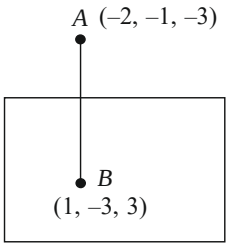
MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

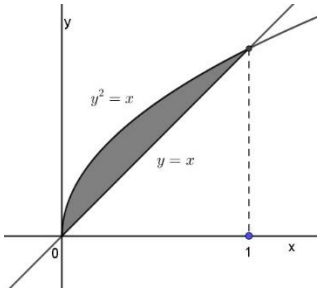
MATHEMATICS (Subject Code–041)

[Paper Code : 65/4/2]

Maximum Marks : 40

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION—A **Question Nos. 1 to 6 carry 2 marks each.**	
Q1.	The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.	**2**
A1.	D.R.'s of normal vector are $\langle 3, -2, 6 \rangle$ Equation of plane is : $3(x-1)-2(y+3)+6(z-3)=0$ $\Rightarrow 3x-2y+6z=27$  	1 1
Q2.	A coin is tossed twice. The following table shows the probability distribution of number of tails : X: 0 | 1 | 2 P(X): K | 6K | 9K (a) Find the value of K. (b) Is the coin tossed biased or unbiased ? Justify your answer.	**2**
A2.	$16K=1$ Let $P(T)=p$ and $P(H)=q$ **(a)** $K=\frac{1}{16}$ **(b)** $P(0)=\frac{1}{16}$, $P(1)=\frac{6}{16}$, $P(2)=\frac{9}{16}$ $q=\frac{1}{4} \Rightarrow p=\frac{3}{4}$ Since $P(H) \neq P(T)$, the coin, so tossed, is biased.	 1 ½ ½

Q3.	(a) If $\lvert \vec{a} \times \vec{b} \rvert^2 + \lvert \vec{a} \cdot \vec{b} \rvert^2 = 400$ and $\lvert \vec{b} \rvert = 5$, then find the value of $\lvert \vec{a} \rvert$. **OR** (b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes.	**2**
A3.	**(a)** $\lvert \vec{a} \rvert^2 \lvert \vec{b} \rvert^2 \sin^2\theta + \lvert \vec{a} \rvert^2 \lvert \vec{b} \rvert^2 \cos^2\theta = 400$ $\lvert \vec{a} \rvert^2 \cdot 25(1) = 400$ $\lvert \vec{a} \rvert^2 = 16$ $\lvert \vec{a} \rvert = 4$	1 ½ ½
	Or **(b)** Let the required vector be $x\hat{i} + x\hat{j} + x\hat{k}$ $\sqrt{3x^2} = 5\sqrt{3}$ $x^2 = 25 \Rightarrow x = \pm 5$ Required vectors are $5\hat{i} + 5\hat{j} + 5\hat{k}$ or $-5\hat{i} - 5\hat{j} - 5\hat{k}$.	 ½ ½ ½ ½
Q4.	Find the general solution of the differential equation $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$.	**2**
A4.	Given differential equation can be written as $\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ Integrating, $\log\lvert\log x\rvert + \log\lvert\tan y\rvert = \log C$ $\tan x \cdot \tan y = C$	1 ½ ½
Q5.	Evaluate : $\int_0^1 x^2 e^x \, dx$	**2**
A5.	$\int_0^1 x^2 e^x dx = x^2 e^x - \int 2xe^x dx]_0^1$ $= x^2 e^x - 2xe^x + 2e^x]_0^1$ $= e - 2$	1 ½ ½

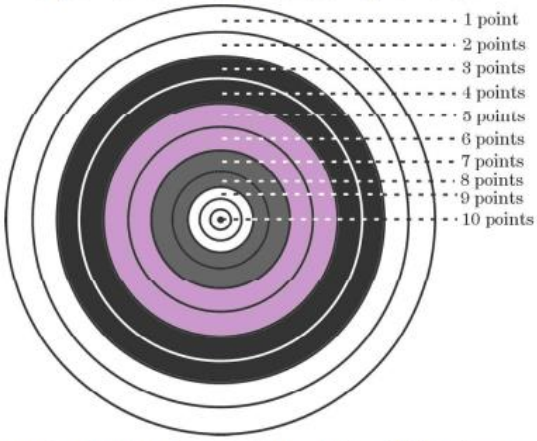
Q6.	There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour.	**2**
A6.	E_1 : Bag 1 is selected E_2 : Bag 2 is selected A : Ball drawn is red in colour $P(A) = P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2)$ $= \frac{1}{2}\times\frac{1}{4} + \frac{1}{2}\times\frac{3}{8}$ $= \frac{5}{16}$	½ 1 ½
	SECTION—B **Question Nos. 7 to 10 carry *3* marks each.**	
Q7.	Using integration, find the area of the region $\{(x, y) : y^2 \le x \le y\}$.	
A7.	 *(for correct figure)* Clearly x coordinates of point of intersection are 0, 1 Required area = $\int_0^1 (\sqrt{x} - x)\,dx$ $= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^2}{2}\right]_0^1$ $= \frac{1}{6}$	1 ½ ½ ½ ½
Q8.	(a) If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. **OR** (b) Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not.	**3**

A8.	**(a)** $l = \cos 60° = \frac{1}{2}$, $n = \cos 45° = \frac{1}{\sqrt{2}}$	½+½
	Now, $l^2 + m^2 + n^2 = 1$	½
	$\Rightarrow \frac{1}{4} + m^2 + \frac{1}{2} = 1$	
	$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$	½
	$\theta = 60°$	
	Required direction cosines are $\left\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}} \right\rangle$	1
	Or	
	(b) Let, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{a}_2 = 4\hat{i} + \hat{j}$	
	$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b}_2 = 5\hat{i} + 2\hat{j} + \hat{k}$	
	$\vec{a}_2 - \vec{a}_1 = 3\hat{i} - \hat{j} - 3\hat{k}$	½
	$\vec{b}_1 \times \vec{b}_2 = -5\hat{i} + 18\hat{j} - 11\hat{k}$	1½
	Here, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -15 - 18 + 33 = 0$	½
	Hence given lines are **not** skew lines.	½
Q9.	(a) Find : $\int \frac{1}{e^x + 1}\, dx$ **OR** (b) Evaluate : $\int_1^4 \{ \lvert x \rvert + \lvert 3 - x \rvert \}\, dx$	**3**
A9.	**(a)** Let, $e^x = t$, $e^x dx = dt$	½
	$\int \frac{1}{e^x + 1} dx = \int \frac{1}{t(t+1)} dt$	½
	$= \int \frac{dt}{t} - \int \frac{dt}{t+1}$	½
	$= \log \lvert t \rvert - \log \lvert t + 1 \rvert + C$	1
	$= \log e^x - \log (e^x + 1) + C$ or $\log \left\lvert \frac{e^x}{1 + e^x} \right\rvert + C$	½

	Or	
	(b) $I = \int_1^4 \{\|x\| + \|3-x\|\}dx$	
	$= \int_1^3 \{\|x\| + \|3-x\|\}dx + \int_3^4 \{\|x\| + \|3-x\|\}dx$	1
	$= \int_1^3 3dx + \int_3^4 (2x-3)dx$	1
	$= \|3x\|_1^3 + \|x^2 - 3x\|_3^4$	½
	$= 6 + 4 = 10$	½
Q10.	If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\frac{\|\vec{a}+\vec{b}\|}{\|\vec{a}-\vec{b}\|} = \cot\left(\frac{\alpha}{2}\right)$.	**3**
A10.	Consider $\frac{\|\vec{a}+\vec{b}\|^2}{\|\vec{a}-\vec{b}\|^2} = \frac{\|\vec{a}\|^2 + \|\vec{b}\|^2 + 2\|\vec{a}\|\|\vec{b}\|\cos\alpha}{\|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\|\vec{a}\|\|\vec{b}\|\cos\alpha}$	1
	$= \frac{2m^2(1+\cos\alpha)}{2m^2(1-\cos\alpha)}$ where $\|\vec{a}\| = \|\vec{b}\| = m$	1
	$= \frac{2\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}$	
	$= \cot^2\left(\frac{\alpha}{2}\right)$	½
	$\therefore \frac{\|\vec{a}+\vec{b}\|}{\|\vec{a}-\vec{b}\|} = \cot\left(\frac{\alpha}{2}\right)$	½
	SECTION—C	
	Question Nos. 11 to 14 carry 4 marks each.	
Q11.	(a) Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$, given that $y(1) = 0$.	**4**
	OR	
	(b) Find the general solution of the differential equation $x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$.	
A11.	**(a)** Given differential equation can be written as $\frac{dy}{dx} + \frac{1}{x}y = \frac{-1}{x(1+x^2)}$	½

	I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$	1
	Solution is $y \cdot x = \int \frac{-1}{1+x^2}dx + C$	1
	$\Rightarrow xy = -\tan^{-1} x + C$	1
	Now $y(1) = 0 \Rightarrow C = \frac{\pi}{4}$	
	$\therefore$ Particular solution is $xy = \frac{\pi}{4} - \tan^{-1} x$	½
	Or	
	(b) Given differential equation can be written as	
	$\frac{dy}{dx} = \frac{2y^4 + 5x^3 y}{xy^3 + x^4}$ …(1)	½
	Let, $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$	½
	Equation (1) becomes $v + x\frac{dv}{dx} = \frac{2v^4 + 5v}{v^3 + 1}$	
	$\Rightarrow x\frac{dv}{dx} = \frac{v^4 + 4v}{v^3 + 1}$	
	$\Rightarrow \frac{v^3 + 1}{v^4 + 4v}dv = \frac{dx}{x}$	1
	$\Rightarrow \int \frac{4v^3 + 4}{v^4 + 4v}dv = 4\int \frac{dx}{x}$	½
	$\Rightarrow \log\lvert v^4 + 4v\rvert = \log(x)^4 + \log C$	1
	$\Rightarrow \log\left\lvert\frac{y^4 + 4yx^3}{x^4}\right\rvert = \log Cx^4$	
	$\Rightarrow\ y^4 + 4yx^3 = Cx^8$	½
Q12.	Evaluate : $\int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}dx$	**4**
A12.	Let, $I = \int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}dx$ …(1)	
	$I = \int_0^{\pi} \frac{\pi - x}{9\sin^2 x + 16\cos^2 x}dx$ (using property) …(2)	1

	Adding equation (1) and (2) $2I = \int_0^{\pi} \frac{\pi}{9\sin^2 x + 16\cos^2 x}\,dx$ $\Rightarrow I = \pi\int_0^{\frac{\pi}{2}} \frac{1}{9\sin^2 x + 16\cos^2 x}\,dx$ (using property) $I = \pi\left[\int_0^{\pi/4} \frac{\sec^2 x}{9\tan^2 x + 16}\,dx + \int_{\pi/4}^{\pi/2} \frac{\operatorname{cosec}^2 x}{9 + 16\cot^2 x}\,dx\right] = \pi\left[I_1 + I_2\right]$ (say) $= \pi\left[\int_0^1 \frac{dt}{9t^2 + 16} - \int_1^0 \frac{dz}{9 + 16z^2}\right]$ $(t = \tan x \text{ in } I_1\ ,\ z = \cot x \text{ in } I_2)$ $= \frac{\pi}{12}\left\{\left[\tan^{-1}\frac{3t}{4}\right]_0^1 - \left[\tan^{-1}\frac{4z}{3}\right]_1^0\right\}$ $= \frac{\pi}{12}\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{4}{3}\right)$ or $\frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$	1 ½ ½ ½ ½
Q13.	Find the equations of the planes passing through the line of intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j}) = 6$ and $\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which are at a distance of 1 unit from the origin.	**4**
A13.	Let the equations of required planes are: $\vec{r}\cdot\left(\hat{i} + 3\hat{j}\right) - 6 + \lambda\left[\vec{r}\cdot\left(3\hat{i} - \hat{j} - 4\hat{k}\right)\right] = 0$ or $\vec{r}\cdot[(1+3\lambda)\hat{i} + (3-\lambda)\hat{j} - 4\lambda\hat{k}] = 6$ for some λ According to given condition $\frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}} = 1$ $36 = 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2$ $\Rightarrow 26\lambda^2 = 26 \Rightarrow \lambda = \pm 1$ Equations of planes are: $\vec{r}\cdot(4\hat{i} + 2\hat{j} - 4\hat{k}) = 6$, $\vec{r}\cdot(-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6$	1 1 1 ½+½

Q14.	In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.  Based on the above information, answer the following questions : If both of them hit the Archery target, then find the probability that (a) exactly one of them earns 10 points. (b) both of them earn 10 points.	**4**
A14.	$P(A) = 0 \cdot 8$, $P(B) = 0 \cdot 9$ (a) P (exactly one of them earns 10 points) = $P(A)P(\bar{B}) + P(\bar{A})P(B)$ $= 0 \cdot 8 \times 0 \cdot 1 + 0 \cdot 2 \times 0 \cdot 9$ $= 0 \cdot 26$ (b) $P(A \cap B) = P(A)P(B)$ $= 0 \cdot 8 \times 0 \cdot 9$ $= 0 \cdot 72$	1½ ½ 1 1