Series : HMJ/5

SET – 1

कोड नं.
Code No. **65/5/1**

रोल नं.
Roll No. | | | | | | | | |

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **15** हैं ।	(I) Please check that this question paper contains **15** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।	(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **36** प्रश्न हैं ।	(III) Please check that this question paper contains **36** questions.
(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**	(IV) **Please write down the Serial Number of the question in the answer-book before attempting it.**
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका में कोई उत्तर नहीं लिखेंगे ।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper comprises* ***four*** *sections – A, B, C and D.* *This question paper carries 36 questions.* ***All*** *questions are compulsory.*

(ii) ***Section A*** *– Question no. 1 to 20 comprises of 20 questions of* ***one*** *mark each.*

(iii) ***Section B*** *– Question no. 21 to 26 comprises of 6 questions of* ***two*** *marks each.*

(iv) ***Section C*** *– Question no. 27 to 32 comprises of 6 questions of* ***four*** *marks each.*

(v) ***Section D*** *– Question no. 33 to 36 comprises of 4 questions of* ***six*** *marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is not permitted.*

Section – A

Question numbers **1** to **10** are multiple choice questions. Select the correct option :

1. If A is a square matrix of order 3, such that A (adj A) = 10 I, then |adj A| is equal to

(a) 1 (b) 10 (c) 100 (d) 101

2. If A is a 3 × 3 matrix such that |A| = 8, then |3A| equals.

(a) 8 (b) 24 (c) 72 (d) 216

P.T.O.

3. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

(a) 25 y (b) 5 y (c) –25 y (d) 15 y

4. $\int x^2 e^{x^3} dx$ equals

(a) $\frac{1}{3} e^{x^3} + C$ (b) $\frac{1}{3} e^{x^4} + C$ (c) $\frac{1}{2} e^{x^3} + C$ (d) $\frac{1}{2} e^{x^2} + C$

5. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

(a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$

6. ABCD is a rhombus whose diagonals intersect at E. Then $\vec{EA} + \vec{EB} + \vec{EC} + \vec{ED}$ equals

(a) $\vec{0}$ (b) $\vec{AD}$ (c) $2\vec{BC}$ (d) $2\vec{AD}$

7. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) – 2 (d) 2

8. The graph of the inequality $2x + 3y > 6$ is

(a) half plane that contains the origin.

(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

(c) whole XOY – plane excluding the points on the line $2x + 3y = 6$.

(d) entire XOY plane.

9. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

10. A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

Fill in the blanks in Questions from **11** to **15**.

11. A relation in a set A is called ______ relation, if each element of A is related to itself.

12. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A = ______.

13. The least value of the function $f(x) = ax + \frac{b}{x}$ $(a > 0, b > 0, x > 0)$ is ______.

14. The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is ______.

OR

The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is ______.

15. The vector equation of a line which passes through the points (3, 4, –7) and (1, – 1, 6) is ______.

OR

The line of shortest distance between two skew lines is ______ to both the lines.

Q. Nos. **16** to **20** are of very short answer type questions.

16. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

If the function f defined as

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

is continuous at $x = 3$, find the value of k.

If $f(x) = x^4 - 10$, then find the approximate value of f(2.1).

OR

Find the slope of the tangent to the curve $y = 2 \sin^2 (3x)$ at $x = \frac{\pi}{6}$.

20. Find the value of $\int_1^4 |x - 5| \, dx$.

Section – B

Q. Nos. **21** to **26** carry **2** marks each.

21. If $f(x) = \frac{4x + 3}{6x - 4}$, $x \neq \frac{2}{3}$, then show that $(fof)(x) = x$, for all $x \neq \frac{2}{3}$. Also, write inverse of f.

OR

Check if the relation R in the set ℝ of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive

22. Find $\int \frac{x}{x^2 + 3x + 2} dx$.

23. *If $x = a \cos \theta$; $y = b \sin \theta$, then find $\frac{d^2y}{dx^2}$.*

OR

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

24. Evaluate $\int_{1}^{2} \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x}\, dx$.

25. Find the value of $\int_{0}^{1} x(1-x)^n\, dx$.

26. Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find $P(A' \cap B')$

Section – C

Q. Nos. 27 to 32 carry 4 marks each.

27. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}(x) = \frac{\pi}{2}$

28. If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$.

29. Solve the differential equation :

$$x \sin\left(\frac{y}{x}\right)\frac{dy}{dx} + x - y \sin\left(\frac{y}{x}\right) = 0$$

Given that $x = 1$ when $y = \frac{\pi}{2}$.

30. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, –1, 4) and C (4, 5, – 1).

31. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹ 100 each and for type B souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as an LPP and solve it graphically.

32. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

Section – D

Q. 33 to 36, carry 6 marks each.

33. *Find the vector and cartesian equations of the line which is perpendicular to the lines with equations*

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4} \text{ and } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and passes through the point (1, 1, 1). Also find the angle between the given lines.

34. Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

OR

Evaluate the following integral as the limit of sums $\int_1^4 (x^2 - x)\, dx$.

35. Find the minimum value of $(ax + by)$, where $xy = c^2$.

36. If a, b, c are p^{th}, q^{th} and r^{th} terms respectively of a G.P, then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

CBSE 2020 ANNUAL EXAMINATION

(Series HMJ/5 Code No. 65/5/1)

MATHEMATICS XII (041)

SECTION A

(Question numbers 01 to 20 carry 1 mark each.)

Q01. (c) $\because A(\text{adj}.A) = |A|I$

$\therefore |A| = 10$

So, $|\text{adj}.A| = |A|^{3-1} = 10^2 = 100$.

Q02. (d) $\because |A| = 8$

$\therefore |3A| = 3^3|A| = 27 \times 8 = 216$.

Q03. (a) $\frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$ and $\frac{d^2y}{dx^2} = 25Ae^{5x} + 25Be^{-5x}$

$\therefore \frac{d^2y}{dx^2} = 25(Ae^{5x} + Be^{-5x}) = 25y$.

Q04. (a) Put $x^3 = y \Rightarrow x^2dx = \frac{dy}{3}$

So, $\int x^2e^{x^3}dx = \frac{1}{3}\int e^y dy = \frac{1}{3}e^y + C$

$\therefore \int x^2e^{x^3}dx = \frac{1}{3}e^{x^3} + C$.

Q05. (c) As $\hat{i}.\hat{k} = |\hat{i}||\hat{k}|\cos\frac{\pi}{2} = 1 \times 1 \times 0 = 0$.

Q06. (a) $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \overrightarrow{EA} + \overrightarrow{EC} - \overrightarrow{EA} - \overrightarrow{EC} = \vec{0}$.

Q07. (a) Re-writing the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$

For the lines to be perpendicular, we must have $1 \times k + 1 \times 2 + (-k) \times (-2) = 0$

$\Rightarrow k = -\frac{2}{3}$.

Q08. (b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

Q09. (c) Let A : the card is a spade and, B : the picked card is a queen.

We have a total of 13 spades and 4 queen cards. Also only one queen is from spade.

$\therefore P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$.

Q10. (d) Here $A=\{4,5,6\}$ and $B=\{1,2,3,4\}$. Note that $A\cap B=\{4\}$.

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{3}{6}+\frac{4}{6}-\frac{1}{6}=1$.

Q11. reflexive

Q12. Consider $2(A+B)+(A-2B)=2\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$

$\Rightarrow 3A=\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$

$\therefore A=\frac{1}{3}\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1/3 & 1/3\\ 2/3 & 1/3\end{bmatrix}$.

Q13. Here $f(x)=ax+\frac{b}{x};\ a>0, b>0, x>0$

$\therefore f'(x)=a-\frac{b}{x^2}$ and, $f''(x)=\frac{2b}{x^3}$

For critical points, $f'(x)=a-\frac{b}{x^2}=0 \quad \Rightarrow x=\sqrt{\frac{b}{a}}$ $\quad\left[\because x>0\ \therefore x\neq-\sqrt{\frac{b}{a}}\right.$

As $f''\left(\sqrt{\frac{b}{a}}\right)=\frac{2b}{\frac{b^{3/2}}{a^{3/2}}}=\frac{2a^{3/2}}{\sqrt{b}}>0$ so, $f(x)$ has least value at $x=\sqrt{\frac{b}{a}}$.

Also, the least value of function is, $f\left(\sqrt{\frac{b}{a}}\right)=a\sqrt{\frac{b}{a}}+\frac{b}{\sqrt{\frac{b}{a}}}=\sqrt{ab}+\sqrt{ab}=2\sqrt{ab}$.

Q14. Re-writing the given D.E. $\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x \quad \therefore P(x)=\left(\frac{2}{x}\right), Q(x)=x$

So, integrating factor is $=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$.

OR

Degree is 2.

Q15. We have $\vec{r}=3\hat{i}+4\hat{j}-7\hat{k}+\lambda\left[(\hat{i}-\hat{j}+6\hat{k})-(3\hat{i}+4\hat{j}-7\hat{k})\right]$

$\therefore \vec{r}=3\hat{i}+4\hat{j}-7\hat{k}+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$ is the required equation.

OR

perpendicular

Q16. $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]=\sin^{-1}\left[\sin\left(-2\pi-\frac{\pi}{8}\right)\right]=\sin^{-1}\left[-\sin\frac{\pi}{8}\right]=-\sin^{-1}\sin\frac{\pi}{8}=-\frac{\pi}{8}$.

Q17. $A^{-1}=\frac{1}{-3-(-4)}\begin{bmatrix}-1 & 4\\ -1 & 3\end{bmatrix}=\begin{bmatrix}-1 & 4\\ -1 & 3\end{bmatrix}$

Q18. As f is continuous at $x=3$ so, we must have $\lim_{x\to3}f(x)=f(3)$

$\Rightarrow \lim_{x\to3}\frac{x^2-9}{x-3}=k$

$\Rightarrow \lim_{x\to 3}(x+3)=k$

$\therefore k=3+3=6$.

Q19. We have $f(x)=x^4-10$

$\Rightarrow f'(x)=4x^3$

As $f(x+h)=f(x)+hf'(x)$, where $h\to 0$

Put $x=2,\ h=0.1$

$\therefore f(2.1)=f(2)+(0.1)f'(2)$

$\Rightarrow f(2.1)=(16-10)+(0.1)(4\times 8)$

Therefore, $f(2.1)=6+3.2=9.2$.

OR

$\frac{dy}{dx}=2\times 2\sin 3x\cos 3x\times 3 = 6\sin 6x$

$\therefore \left.\frac{dy}{dx}\right]_{\text{at } x=\pi/6}=6\sin\pi=6\times 0=0$.

Q20. $\int_1^4 |x-5|dx = -\int_1^4 (x-5)dx = -\frac{1}{2}\left[(x-5)^2\right]_1^4$

$\Rightarrow \quad = -\frac{1}{2}[1-16]=\frac{15}{2}$.

SECTION B

(Question numbers 21 to 26 carry 2 marks each.)

Q21. Given $f(x)=\frac{4x+3}{6x-4},\ x\neq\frac{2}{3}$

Now $fof(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$

$\Rightarrow fof(x)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$

Also let $y=f(x)=\frac{4x+3}{6x-4}$

$\Rightarrow 6xy-4y=4x+3$

$\Rightarrow 6xy-4x=3+4y$

$\Rightarrow x=\frac{3+4y}{6y-4}$

So, $f^{-1}=\frac{3+4y}{6y-4}$ i.e., $f^{-1}(x)=\frac{3+4x}{6x-4},\ x\neq\frac{2}{3}$.

OR

We have $R = \{(a,b) : a < b\}$ where $a,b \in \mathbf{R}$.

(i) Symmetry : Observe that $1 < 2$ is true but $2 < 1$ is not true.
That is, $(1,2) \in R$ but $(2,1) \notin R$ so, R is not symmetric.

(ii) Transitivity : Observe that if $a < b$ and $b < c$ are both true then, $a < c$ is also true, for all real numbers a, b, c.
That is, $(a, b) \in R$ and $(b,c) \in R$ implies $(a,c) \in R$ so, R is transitive.

Q22. $$\int \frac{x\,dx}{x^2+3x+2} = \frac{1}{2}\int \frac{(2x+3)dx}{x^2+3x+2} - \frac{3}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow \int \frac{x\,dx}{x^2+3x+2} = \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\times\frac{1}{2\times\frac{1}{2}}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right| + C$$

$$\therefore \int \frac{xdx}{x^2+3x+2} = \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C.$$

Q23. Here $x = a\cos\theta,\ y = b\sin\theta$

$$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta,\ \frac{dy}{d\theta} = b\cos\theta$$

So, $\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = b\cos\theta\times\frac{1}{-a\sin\theta} = -\frac{b}{a}\cot\theta$

Now $\frac{d^2y}{dx^2} = \frac{b}{a}\text{cosec}^2\theta\times\frac{d\theta}{dx} = \frac{b}{a}\text{cosec}^2\theta\times\frac{1}{-a\sin\theta} = -\frac{b}{a^2}\text{cosec}^3\theta$.

OR

Let $y = \sin^2 x,\ z = e^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = 2\sin x\cos x = \sin 2x,\ \frac{dz}{dx} = e^{\cos x}\times(-\sin x)$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx}\times\frac{dx}{dz} = 2\sin x\cos x\times\frac{1}{e^{\cos x}\times(-\sin x)}$$

That is, $\frac{dy}{dz} = -\frac{2\cos x}{e^{\cos x}}$.

Q24. Consider $\int\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx = \frac{1}{2}\int\left[\frac{2}{y}-\frac{2}{y^2}\right]e^y dy = \int\left[\frac{1}{y}+\frac{-1}{y^2}\right]e^y dy = \left[\frac{1}{y}\times e^y\right] = \frac{e^{2x}}{2x}$

[Put $2x = y \Rightarrow dx = \frac{dy}{2}$

So, $\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx = \left[\frac{e^{2x}}{2x}\right]_1^2 = \frac{1}{2}\left[\frac{e^4}{2}-\frac{e^2}{1}\right] = \frac{e^2}{2}\left[\frac{e^2}{2}-1\right]$.

Q25. Let $I = \int_0^1 x(1-x)^n dx$

$$\Rightarrow I = \int_0^1 (1-x)[1-(1-x)]^n dx$$

$$\Rightarrow I=\int_0^1 (1-x)x^n dx$$

$$\Rightarrow I=\int_0^1 (x^n - x^{n+1})dx$$

$$\Rightarrow I=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$$

$$\Rightarrow I=\left[\frac{1}{n+1}-\frac{1}{n+2}\right]-[0-0]$$

$$\therefore I=\frac{1}{(n+1)(n+2)}.$$

Q26. As A and B are independent events so, A' and B' are also independent.

$\therefore P(A'\cap B')=P(A')\times P(B')$

$\Rightarrow P(A'\cap B')=[1-P(A)]\times[1-P(B)]$

$\Rightarrow P(A'\cap B')=[1-0.3]\times[1-0.6]=0.7\times 0.4$

Therefore, $P(A'\cap B')=0.28$.

SECTION C

(Question numbers 27 to 32 carry 4 marks each.)

Q27. We have $\sin^{-1}(1-x)-2\sin^{-1}x=\dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x)=\pi/2+2\sin^{-1}x$

$\Rightarrow \sin\left[\sin^{-1}(1-x)\right]=\sin\left[\pi/2+2\sin^{-1}x\right]$

$\Rightarrow 1-x=\cos(2\sin^{-1}x)$

$\Rightarrow 1-x=1-2[\sin(\sin^{-1}x)]^2$ (By using $\cos 2\theta=1-2\sin^2\theta$

$\Rightarrow 1-x=1-2x^2$ $\Rightarrow 2x^2-x=0$

$\Rightarrow x(2x-1)=0$

$\Rightarrow x=0, \dfrac{1}{2}$

$\because x=\dfrac{1}{2}$ doesn't satisfy the given equation.

$\therefore x=0$ is the required solution.

Q28. Here $y=(\log x)^x+x^{\log x}$

$$\Rightarrow y=e^{\log(\log x)^x}+e^{\log x^{\log x}}$$

$$\Rightarrow y=e^{x\log(\log x)}+e^{\log x\log x}=e^{x\log(\log x)}+e^{(\log x)^2}$$

$$\therefore \frac{dy}{dx}=e^{x\log(\log x)}\left\{x\times\frac{1}{\log x}\times\frac{1}{x}+\log(\log x)\times 1\right\}+e^{(\log x)^2}\left\{2\log x\times\frac{1}{x}\right\}$$

$$\therefore \frac{dy}{dx}=(\log x)^x\left\{\frac{1}{\log x}+\log(\log x)\right\}+x^{\log x}\left\{\frac{2\log x}{x}\right\}.$$

Q29. $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right)+x-y\sin\left(\dfrac{y}{x}\right)=0$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} \quad ...(i)$$

Consider $f(x, y) = \frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right)$

On putting $x = \lambda x,\ y = \lambda y$, we get : $f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} - \text{cosec}\left(\frac{\lambda y}{\lambda x}\right) = \frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right) = f(x, y)$

So, it is homogeneous.
Now put $y = vx$ in (i).

On differentiating w. r. t. x both the sides, we get : $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So by (i), we have : $v + x\frac{dv}{dx} = \frac{vx}{x} - \frac{1}{\sin(vx/x)}$

$$\Rightarrow -\int \sin v\, dv = \int \frac{dx}{x}$$

$$\Rightarrow \cos v = \log|x| + k$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + k$$

It is given that x = 1 when $y = \frac{\pi}{2}$, so $\cos\left(\frac{\pi/2}{1}\right) = k - \log|1|$

$$\Rightarrow k = 0$$

$\therefore\ \cos\left(\frac{y}{x}\right) = \log|x|$ is the required solution.

Q30. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

The diagonals of parallelogram are given as $\vec{p} = \vec{b} + \vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k},\ \vec{q} = \vec{b} - \vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$

$\therefore \hat{p} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9 + 36 + 4}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$ and, $\hat{q} = \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{1 + 4 + 64}} = \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{69}}$

Note that, the second diagonal $\vec{q}$ can be taken as $\vec{q} = \vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 8\hat{k}$ as well. In that case, $\hat{q} = \frac{-\hat{i} - 2\hat{j} + 8\hat{k}}{\sqrt{1 + 4 + 64}} = -\frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{69}}$.

OR

We've $\overrightarrow{AB} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} - 3\hat{j} + \hat{k}$,

$\overrightarrow{AC} = (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 3\hat{i} + 3\hat{j} - 4\hat{k}$.

Also $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} = 9\hat{i} + 7\hat{j} + 12\hat{k}$

So, $\text{ar}(ABC) = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\Rightarrow \text{ar}(ABC) = \frac{1}{2}\sqrt{81 + 49 + 144}$$

$\therefore ar(ABC) = \dfrac{\sqrt{274}}{2}$ Sq. units.

Q31. Let the number of Souvenirs of type A and type B be x and y, respectively.

To maximize : $Z = ₹(100x + 120y)$

Subject to constraints :

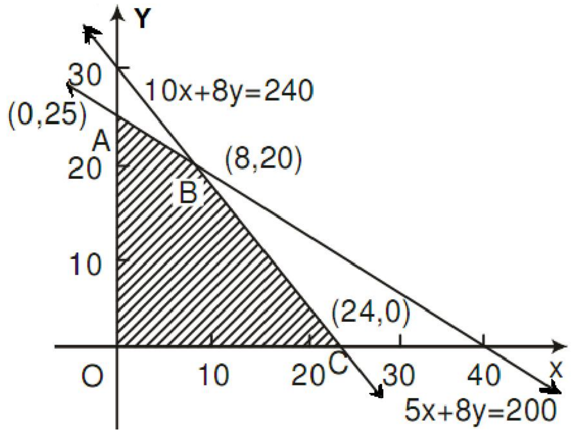
$x \geq 0,$

$y \geq 0,$

$5x + 8y \leq 200,$

$10x + 8y \leq 240$

Corner Points	Value of Z (in ₹)
A(0, 25)	3000
B(8, 20)	3200 ← Maximum
C(24, 0)	2400



Hence maximum profit of ₹3200 is obtained when 8 souvenirs of type A and 20 souvenirs of type B is manufactured.

Q32. Here rotten apples are = 3 and, fresh apples are = 7.

Total no. of apples = 10.

Let X : no. of rotten apples.

So X can take values 0, 1, 2, 3.

Let E : getting a rotten apple.

$P(E) = \dfrac{3}{10}, \; P(E') = \dfrac{7}{10}.$

So, $P(X = 0) = P(E')P(E')P(E') = \dfrac{343}{1000}, \; P(X = 1) = 3P(E)P(E')P(E') = \dfrac{441}{1000},$

$P(X = 2) = 3P(E)P(E)P(E') = \dfrac{189}{1000}, \; P(X = 3) = P(E)P(E)P(E) = \dfrac{27}{1000}$

The probability distribution table is :

X	0	1	2	3
P(X)	$\dfrac{343}{1000}$	$\dfrac{441}{1000}$	$\dfrac{189}{1000}$	$\dfrac{27}{1000}$

Now mean, $\mu = \sum X \; P(X) = 0 \times \dfrac{343}{1000} + 1 \times \dfrac{441}{1000} + 2 \times \dfrac{189}{1000} + 3 \times \dfrac{27}{1000}$

$\Rightarrow \mu = \dfrac{900}{1000}$ or, $\dfrac{9}{10}.$

OR

Let A : getting type B ghee, E_1 : getting ghee from shop X, E_2 : getting ghee from shop Y.

$\therefore P(E_1) = \dfrac{1}{2}, \; P(E_2) = \dfrac{1}{2},$

$P(A|E_1)=\frac{40}{70}=\frac{4}{7}$, $P(A|E_2)=\frac{60}{110}=\frac{6}{11}$

Using Bayes' Theorem, $P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$\Rightarrow P(E_2|A)=\frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}}=\frac{\frac{6}{11}}{\frac{4}{7}+\frac{6}{11}}=\frac{42}{44+42}=\frac{42}{86}$$

$\therefore P(E_2|A)=\frac{21}{43}$.

SECTION D

(Question numbers 33 to 36 carry 6 marks each.)

Q33. Let the d.r.'s of the required line L (*say*) be a, b, c.

Since L is perpendicular to $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and, $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

So, using $a_1a_2+b_1b_2+c_1c_2=0$ we get :

$a+2b+4c=0\ldots(i)$

$2a+3b+4c=0\ldots(ii)$

Solving (i) and (ii), $\frac{a}{8-12}=\frac{b}{8-4}=\frac{c}{3-4}$ i.e., $\frac{a}{-4}=\frac{b}{4}=\frac{c}{-1}$.

Hence the d.r.'s of line L are –4, 4, –1.

The required vector and Cartesian equations of the line L are respectively

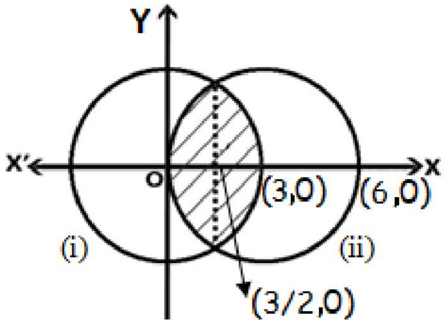
$\vec{r}=\hat{i}+\hat{j}+\hat{k}+\lambda(-4\hat{i}+4\hat{j}-\hat{k})$ and $\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$.

Let θ be the angle between given lines.

So, $\cos\theta=\frac{|1\times2+2\times3+4\times4|}{\sqrt{1+4+16}\sqrt{4+9+16}}=\frac{24}{\sqrt{21}\sqrt{29}}$

$\therefore \theta=\cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$.

Q34. Consider the diagram



Here $x^2+y^2=9\ldots(i)$ and $(x-3)^2+y^2=9\ldots(ii)$

Centre of (i) and (ii) are at (0, 0) and (3, 0) respectively.

Solving (i) and (ii), we get : $9-6x=0 \qquad \Rightarrow x=\frac{3}{2}$

$$\text{Required area}=2\left\{\int_0^{3/2}\sqrt{9-(x-3)^2}\,dx+\int_{3/2}^{3}\sqrt{9-x^2}\,dx\right\}$$

$$\Rightarrow \quad =2\left\{\left[\frac{x-3}{2}\sqrt{9-(x-3)^2}+\frac{9}{2}\sin^{-1}\frac{x-3}{3}\right]_0^{3/2}+\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3}\right\}$$

$$\Rightarrow \quad =2\left\{\begin{array}{l}\left[-\frac{3}{4}\sqrt{9-\frac{9}{4}}+\frac{9}{2}\sin^{-1}\left(-\frac{1}{2}\right)\right]-\left[0+\frac{9}{2}\sin^{-1}(-1)\right]\\ \qquad +\left[0+\frac{9}{2}\sin^{-1}1\right]-\left[\frac{3}{4}\sqrt{9-\frac{9}{4}}+\frac{9}{2}\sin^{-1}\frac{1}{2}\right]\end{array}\right\}$$

$$\Rightarrow \quad =2\left\{-\frac{9\sqrt{3}}{8}-\frac{9}{2}\times\frac{\pi}{6}+\frac{9}{2}\times\frac{\pi}{2}+\frac{9}{2}\times\frac{\pi}{2}-\frac{9\sqrt{3}}{8}-\frac{9}{2}\times\frac{\pi}{6}\right\}$$

$$\Rightarrow \quad =2\left\{-2\times\frac{9\sqrt{3}}{8}-9\times\frac{\pi}{6}+\frac{9}{2}\times\pi\right\}$$

$$\Rightarrow \quad =2\left\{3\pi-\frac{9\sqrt{3}}{4}\right\}\text{ Sq. units}$$

OR

$$\text{Let } I=\int_1^4 (x^2-x)dx$$

We know $\int_a^b f(x)\,dx=\lim_{n\to\infty} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$,

As $n\to\infty,\ h\to 0 \Rightarrow nh=b-a=4-1=3$

$$\therefore \int_a^b f(x)dx=\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh) \qquad \ldots(i)$$

Here $f(x)=x^2-x,\ a=1,\ b=4$.

$\therefore f(a+rh)=(a+rh)^2-(a+rh)$

$\Rightarrow f(1+rh)=(1+rh)^2-(1+rh)$

$\Rightarrow f(1+rh)=r^2h^2+rh$.

By using (i), $\int_1^4 (x^2-x)dx=\lim_{n\to\infty} h\sum_{r=0}^{n-1}\left[r^2h^2+rh\right]$

$$\Rightarrow I=\lim_{n\to\infty} h\left\{h^2\sum_{r=0}^{n-1} r^2+h\sum_{r=0}^{n-1} r\right\}$$

$$\Rightarrow I=\lim_{n\to\infty} h\left\{h^2\times\frac{n(n-1)(2n-1)}{6}+h\frac{n(n-1)}{2}\right\}$$

$$\Rightarrow I=\lim_{n\to\infty}\left\{\frac{nh(nh-h)(2nh-h)}{6}+\frac{nh(nh-h)}{2}\right\}$$

$$\Rightarrow I=\frac{3(3-0)(6-0)}{6}+\frac{3(3-0)}{2}$$

$$\Rightarrow I=9+\frac{9}{2}=\frac{27}{2}.$$

Q35. Given $xy=c^2 ...(i)$

Let $S=(ax+by)$

$$\Rightarrow S=ax+\frac{bc^2}{x}$$

$$\Rightarrow \frac{dS}{dx}=a-\frac{bc^2}{x^2}$$

Also, $\frac{d^2S}{dx^2}=\frac{2bc^2}{x^3}$

For local points of maxima and/or minima, we have : $\frac{dS}{dx}=0$

$$\Rightarrow a-\frac{bc^2}{x^2}=0$$

$$\Rightarrow x=c\sqrt{\frac{b}{a}}$$

$$\because \left.\frac{d^2S}{dx^2}\right]_{\text{at } x=c\sqrt{\frac{b}{a}}}=\frac{2bc^2}{c^3\left(\frac{b}{a}\right)^{3/2}}>0$$

$\therefore$ S is minimum at $x=c\sqrt{\frac{b}{a}}$.

Also, minimum value of $S=ax+by=2ax$ $\left[\begin{array}{l}\because x=c\sqrt{\frac{b}{a}}\Rightarrow c^2=\frac{ax^2}{b}\\ \text{Replacing value of } c^2 \text{ in (i), we get } ax=by\end{array}\right]$

That is, $S=2ac\sqrt{\frac{b}{a}}$

$\therefore S=2c\sqrt{ab}$.

Q36. Given that a, b, c are p^{th}, q^{th} and r^{th} terms of a G.P. then,

$A_p=AR^{p-1}=a, A_q=AR^{q-1}=b, A_r=AR^{r-1}=c$, where A and R are the 1^{st} term and common ratio of the geometric progression respectively.

Consider LHS : Let $\Delta=\begin{vmatrix}\log a & p & 1\\ \log b & q & 1\\ \log c & r & 1\end{vmatrix}$

$$\Rightarrow \Delta=\begin{vmatrix}\log\left[AR^{p-1}\right] & p & 1\\ \log\left[AR^{q-1}\right] & q & 1\\ \log\left[AR^{r-1}\right] & r & 1\end{vmatrix} \qquad \left[\begin{array}{l}\because \log(mn)=\log m+\log n,\\ \log(m)^n=n\log m\end{array}\right.$$

$$\therefore \Delta=\begin{vmatrix}\log A+(p-1)\log R & p & 1\\ \log A+(q-1)\log R & q & 1\\ \log A+(r-1)\log R & r & 1\end{vmatrix}$$

By $C_1 \to C_1 - (\log A)C_3$

$$\Rightarrow \Delta = \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$$

Taking log R common from C_1

$$\Rightarrow \Delta = \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

By $C_1 \to C_1 + C_3$

$$\Rightarrow \Delta = \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix}$$

Since C_1 and C_2 are identical, $\therefore \Delta = 0 = \text{RHS}$.

OR

Here $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 3(-2) + 5(1) = -1 \neq 0 \quad \therefore A^{-1} \text{ exists.}$$

Consider A_{ij} be the cofactors of element a_{ij} of A.

$A_{11} = 0,\ A_{12} = 2,\ A_{13} = 1,\ A_{21} = -1,\ A_{22} = -9,\ A_{23} = -5,\ A_{31} = 2,\ A_{32} = 23,\ A_{33} = 13$

$$\therefore \text{adj.}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \times \text{adj.}A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \ldots\text{(i)}$$

Now consider the equations,

$2x - 3y + 5z = 11,$

$3x + 2y - 4z = -5,$

$x + y - 2z = -3$

Let $M = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ N = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\because MX = N$

$\Rightarrow X = M^{-1}N = A^{-1}N \qquad (\because A = M$

By (i), $X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By equality of matrices, we get : $x = 1,\ y = 2,\ z = 3$.