Series SSJ/1 – Set 4

CBSE
Class XII Mathematics
Board Paper
Term 1 – 2021

Time: 90 minutes **Total Marks: 40**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This question paper contains **50** questions out of which **40** questions are to be attempted as per instructions. All questions carry equal marks.*

(ii) *The Question Paper consists of three sections- Section A, Section B and Section C.*

(iii) *Section A contains **20** questions. Attempt any **16** questions from questions no. **1** to **20**.*

(iv) *Section B also contains **20** questions. Attempt any **16** questions from questions no. **21** to **40**.*

(v) *Section C contains **10** questions. Attempt any **8** questions from Q. No. **41** to **50***

(vi) *There is only **one** correct option for every multiple choice question (MCQ). Marks will not be awarded for answering more than one option.*

(vii) *There is **no** negative marking.*

SECTION A

In this section, attempt any 16 questions out of questions no. 1 to 20.

1. A relation R is defined on N. Which of the following is the reflexive relation?

(a) $R = \{(x, y): x > y; x, y \in N\}$

(b) $R = \{(x, y): x + y = 10; x, y \in N)\}$

(c) $R = \{(x, y)$: xy is the square number; $x, y \in N\}$

(d) $R = \{(x, y): x + 4y = 10; x, y \in N\}$

2. The function $f: R \to R$ defined by $f(x) = 4 + 3 \cos x$ is:

(a) bijective

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

3. If $y = \cot^{-1} x, x < 0$, then:

(a) $\frac{\pi}{2} < y \le \pi$

(b) $\frac{\pi}{2} < y < \pi$

(c) $-\frac{\pi}{2} < y < 0$

(d) $-\frac{\pi}{2} \le y < 0$

4. The number of functions defined from {1, 2, 3, 4, 5} ⟶ {a, b} which are one-one is:

(a) 5
(b) 3
(c) 2
(d) 0

5. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then (A − 2I) (A − 3I) is equal to:

(a) A
(b) I
(c) 5I
(d) O

6. If P is a 3 x 3 matrix such that P′ = 2P + I, where P′ is the transpose of P, then:

(a) P = I
(b) P = −I
(c) P = 2I
(d) P = −2I

7. If order of matrix A is 2 x 3, of matrix B is 3 x 2, and of matrix C is 3 x 3, then which one of the following is not defined?

(a) C (A + B′)
(b) C (A + B′)′
(c) BAC
(d) CB + A'

8. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is:

(a) ± 1
(b) ± 2
(c) $\pm\sqrt{5}$
(d) $\pm\sqrt{7}$

9. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is:

(a) 3
(b) 5
(c) 7
(d) 9

10. The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is:

(a) $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$, then A^{-1}:

(a) is A
(b) is (-A)
(c) is A^2
(d) does not exist

12. If the function $f(x) = \begin{cases} 3x - 8, \text{if} & x \le 5 \\ 2k, \text{if} & x > 5 \end{cases}$ is continuous, then the value of k is:

(a) $\frac{2}{7}$

(b) $\frac{7}{2}$

(c) $\frac{3}{7}$

(d) $\frac{4}{7}$

13. The function f(x) = [x], where lx] is the greatest integer function that is less than or equal to x, is continuous at:

(a) 4

(b) −2

(c) 1.5

(d) 1

14. If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{2e^{2x}}{1+e^{4x}}$

(b) $\frac{1}{1+e^{4x}}$

(c) $\frac{2}{e^{2x}+e^{-2x}}$

(d) $\frac{1}{e^{2x}-e^{-2x}}$

15. If $y^2 (2 - x) = x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to:

(a) 2

(b) −2

(c) 3

(d) $-\frac{3}{2}$

16. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is:

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) 0

17. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing is:

(a) $(-\infty, -4) \cup (0, \infty)$

(b) $(-\infty, 4)$

(c) $(-4, 0)$

(d) $(-\infty, 0) \cup (4, \infty)$

18. The value of x for which $(x - x^2)$ is maximum, is:

(a) $\frac{3}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

19. If the corner points of the feasible region of an LPP are (0, 3), (3, 2) and (0, 5), then the minimum value of $Z = 11x + 7y$ is:

(a) 21
(b) 33
(c) 14
(d) 35

20. The number of solutions of the system of inequations $x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$ is:

(a) 0
(b) 2
(c) finite
(d) infinite

SECTION B

In this section, attempt any **16** questions out of questions no. **21** to **40**.

21. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is:

(a) 0
(b) 1
(c) 2
(d) 3

22. Let f: R $\longrightarrow$ R be defined by $f(x) = \frac{1}{x}$, for all $x \in R$. Then, f is:

(a) one-one
(b) onto
(c) bijective
(d) not defined

23. The function f: N ⟶ N is defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$

The function f is:

(a) bijective

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

24. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is:

(a) $-\frac{3\pi}{5}$

(b) $-\frac{\pi}{10}$

(c) $\frac{3\pi}{5}$

(d) $\frac{\pi}{10}$

25. If $\sin^{-1} x > \cos^{-1} x$, then x should lie in the interval:

(a) $\left(-1, -\frac{1}{\sqrt{2}}\right)$

(b) $\left(0, \frac{1}{\sqrt{2}}\right)$

(c) $\left(\frac{1}{\sqrt{2}}, 1\right)$

(d) $\left(-\frac{1}{\sqrt{2}}, 0\right)$

26. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and A + A′ = I, then the value of α is:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) Π

(d) $\frac{3\pi}{2}$

27. The determinant $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to:

(a) $k(3y + k^2)$
(b) $3y + k^3$
(c) $3y + k^2$
(d) $k^2(3y + k)$

28. If A = $\begin{vmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix}$ is the adjoint of a square matrix B, then B^{-1} is equal to:

(a) $\pm A$
(b) $\pm\sqrt{2}A$
(c) $\pm\frac{1}{\sqrt{2}}B$
(d) $\pm\frac{1}{\sqrt{2}}A$

29. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then $A^5 - A^4 - A^3 + A^2$ is equal to

(a) 2A
(b) 2A
(c) 4A
(d) O

30. If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is equal to:

(a) $-y$
(b) y
(c) x
(d) $-x$

31. If $x = t^2 + 1$, $y = 2at$, then $\frac{d^2y}{dx^2}$ at t = a is:

(a) $-\frac{1}{a}$
(b) $-\frac{1}{2a^2}$
(c) $\frac{1}{2a^2}$
(d) 0

32. The function $f(x)=\begin{cases}x^2 & \text{, for } x<1\\ 2-x, & \text{for } x\geq 1\end{cases}$ is:

(a) not differentiable at x = 1
(b) differentiable at x = 1
(c) not continuous at x = 1
(d) neither continuous nor differentiable at x = 1

33. The curve $x^2 - xy + y^2 = 27$ has tangents parallel to x-axis at:
(a) (3, 6) and (−3, −6)
(b) (3, −6) and (−3, 6)
(c) (−3, −6) and (3, −6)
(d) (−3, 6) and (−3, −6)

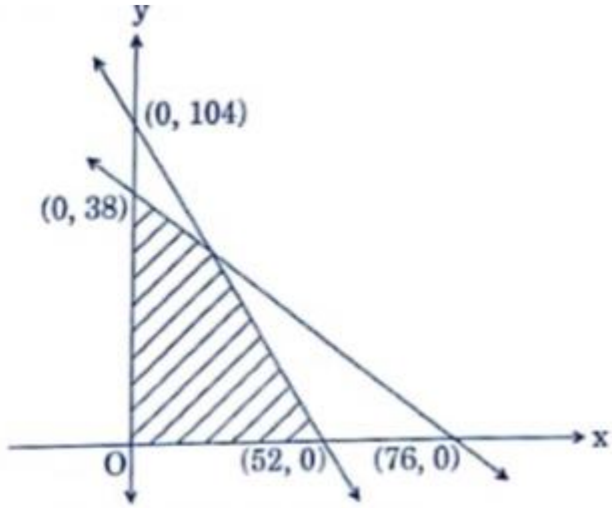
34. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is:
(a) 20 sq cm
(b) 25 sq cm
(c) 10 sq cm
(d) 30 sq cm

35. The function (x − sin x) decreases for:
(a) all x
(b) $x<\frac{\pi}{2}$
(c) $0<x<\frac{\pi}{4}$
(d) no value of x

36. If θ is the angle of intersection between the curves $y^2 = 4ax$ and $ay = 2x^2$ at (a, 2a), then the value of $\tan\theta$ is:
(a) $\frac{3}{5}$
(b) $\frac{2}{3}$
(c) $\frac{3}{4}$
(d) $\frac{2}{5}$

37. The maximum value of Z = 3x + 4y subject to the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 1$ is:
(a) 7
(b) 4
(c) 3
(d) 10

38. The feasible region of an LPP is given in the following figure:



Then, the constraints of the LPP are $x \geq 0$, $y \geq 0$ and

(a) $2x + y \leq 52$ and $x + 2y \leq 76$
(b) $2x + y \leq 104$ and $x + 2y \leq 76$
(c) $x + 2y \leq 104$ and $2x + y \leq 76$
(d) $x + 2y \leq 104$ and $2x + y \leq 38$

39. If the minimum value of an objective function $Z = ax + by$ occurs at two points (3, 4) and (4, 3), then:

(a) $a + b = 0$
(b) $a = b$
(c) $3a = b$
(d) $a = 3b$

40. For the following LPP

Maximise $Z = 3x + 4y$

Subject to constraints

$x - y \geq -1$, $x \leq 3$

$x \geq 0$, $y \geq 0$

The maximum value is:

(a) 0
(b) 4
(c) 25
(d) 30

SECTION C

In this section, attempt any 8 questions out of questions no. 41 – 50. $8 \times 1 = 8$

41. A relation R is defined on Z as:
aRb if and only if $a^2 - 7ab + 6b^2 = 0$.
Then, R is:
(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric

42. The value of $\begin{vmatrix} 1! & 2! & 3! \\ 2\,2! & 3\,3! & 4\,4! \\ 3! & 4! & 5! \end{vmatrix}$ is:
(a) 12
(b) –12
(c) 24
(d) –24

43. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then:
(a) $a = 1 = b$
(b) $a = \cos 2\theta$, $b = \sin 2\theta$
(c) $a = \sin 2\theta$, $b = \cos 2\theta$
(d) $a = \cos\theta$, $b = \sin\theta$

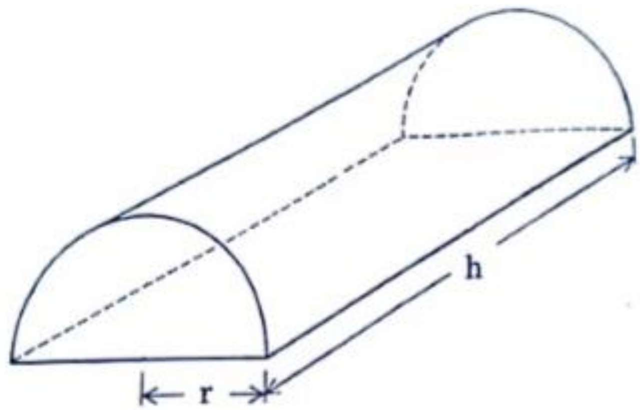
44. The normal to the curve $3y = 6x - 5x^3$ at the point $\left(1, \frac{1}{3}\right)$ passes through the point:
(a) (3, 1)
(b) (3, 2)
(c) (2, 3)
(d) (1, 1)

45. If $y = \sin(2 \sin^{-1}x)$, then $(1 - x^2)y_2$ is equal to:
(a) $-xy_1 + 4y$
(b) $-xy_1 - 4y$
(c) $xy_1 - 4y$
(d) $xy_1 + 4y$

Case Study

Some young entrepreneurs started an industry "young achievers" for casting metal into various shapes. They put up an advertisement online stating the same and expecting order to cast metal for toys sculptures, decorative pieces and more.

A group of friends wanted to make innovative toys and hence contacted the "young achievers" to order them to cast metal into solid half cylinders with a rectangular base and semi-circular ends.



Based on the above information, answer the following questions:

46. The volume (V) of the casted half cylinder will be:

(a) $\Pi r^2 h$

(b) $\frac{1}{3}\pi r^2 h$

(c) $\frac{1}{2}\pi r^2 h$

(d) $\Pi r^2(r + h)$

47. The total surface area (S) of the casted half cylinder will be:

(a) $\pi rh + 2\pi r^2 + rh$

(b) $\pi rh + \pi r^2 + 2rh$

(c) $2\pi rh + \pi r^2 + 2rh$

(d) $\pi rh + \pi r^2 + rh$

48. The total surface area S can be expressed in terms of V and r as:

(a) $2\pi r + \frac{2V(\pi + 2)}{\pi r}$

(b) $\pi r + \frac{2V}{\pi r}$

(c) $\pi r^2 + \frac{2V(\pi + 2)}{\pi r}$

(d) $2\pi r^2 + \frac{2V(\pi + 2)}{\pi r}$

49. For the given half-cylinder of volume V, the total surface area S is minimum, when:

(a) $(\pi + 2)V = \pi^2 r^3$

(b) $(\pi + 2)V = \pi^2 r^2$

(c) $2(\pi + 2)V = \pi^2 r^3$

(d) $(\pi + 2)V = \pi^2 r$

50. The ratio h : 2r for S to be minimum will be equal to:

(a) $2\pi : \pi + 2$

(b) $2\pi : \pi + 1$

(c) $\pi : \pi + 1$

(d) $\pi : \pi + 2$

Solution

SECTION A

1. Correct Option: (c)
When $x \in N$, x^2 is a square number
So, $(x, x) \in R$ for all $x \in N$.
Therefore, R is reflexive.

2. Correct Option: (d)
Given: $f(x) = 4 + 3 \cos x$
Since, $\cos\frac{\pi}{2} = \cos\left(-\frac{\pi}{2}\right)$

$\Rightarrow 4 + 3\cos\frac{\pi}{2} = 4 + 3\cos\left(-\frac{\pi}{2}\right)$

$\Rightarrow f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$

But $\frac{\pi}{2} \neq -\frac{\pi}{2}$
So, f is not one-one.
Range of cos x is $[-1, 1]$.
$\Rightarrow -1 \leq \cos x \leq 1$
$\Rightarrow -3 \leq 3\cos x \leq 3$
$\Rightarrow 1 \leq 4 + 3\cos x \leq 7$
$\Rightarrow 1 \leq f(x) \leq 7$
So, the range of f is $[1, 7]$.
Thus, f is not onto.
Hence, f is neither one-one nor onto.

3. Correct Option: (b)
Given: $y = \cot^{-1} x$, $x < 0$
Therefore, $x = \cot y$
As $x < 0$, $\cot y < 0$
Now, $\cot y < 0$ when $\frac{\pi}{2} < y < \pi$

4. Correct Option: (d)
If we consider one-one function, only two elements of the set {1, 2, 3, 4, 5} can have images.
Therefore, there can't be a one-one function from $\{1, 2, 3, 4, 5\} \rightarrow \{a, b\}$.
Hence, the number of one-one functions is 0.

5. Correct Option: (d)

Given: $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$(A-2I)(A-3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= O$

6. Correct Option: (b)

Given: P′ = 2P + I ... (i)

Therefore, (P′)′ = (2P + I)′

i.e. P = 2P′ + I

i.e. P = 2(2P + I) + I ... From (i)

Therefore, P = –I

7. Correct Option: (a)

A + B′ is a matrix of order 2 x 3

Since C is a matrix of order 3 x 3

So, C(A + B′) is not defined.

8. Correct Option: (d)

Given: $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$

Since, $\det\left(A^n\right) = \left(\det(A)\right)^n$

$\Rightarrow \left|A^3\right| = |A|^3$

$\Rightarrow \left(\alpha^2 - 4\right)^3 = 27$

$\Rightarrow \alpha^2 - 4 = 3$

$\Rightarrow \alpha = \pm\sqrt{7}$

9. Correct Option: (d)

Given: $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow 5(-2x+18)-3(14+27)-1(-42-9x)=0$
$\Rightarrow -10x+90-42-81+42+9x=0$
$\Rightarrow x=9$

10. Correct Option: (b)

Inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

11. Correct Option: (a)

Given: $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$

Cofactor matrix of A is $\begin{bmatrix} -1 & 0 & -59 \\ 0 & -1 & -69 \\ 0 & 0 & 1 \end{bmatrix}$

Adjoint of A is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -59 & -69 & 1 \end{bmatrix}$

So, the inverse of A is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$

12. Correct Option: (b)

Given: $f(x)=\begin{cases} 3x-8, & \text{if } x \le 5 \\ 2k, & \text{if } x > 5 \end{cases}$

As f is continuous, so it will be continuous at 5.

$\Rightarrow \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$
$\Rightarrow \lim_{x\to 5}(3x-8) = \lim_{x\to 5} 2k$
$\Rightarrow 2k = 15-8$
$\Rightarrow k = \frac{7}{2}$

13. Correct Option: (c)

The function [x] is continuous at non-integer values.
Hence, f(x) is continuous at 1.5

14. Correct Option: (a)

Given: $y = \tan^{-1}(e^{2x})$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+\left(e^{2x}\right)^2} \times \frac{d}{dx}\left(e^{2x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{2x}}{1+e^{4x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{2x}}{1+e^{4x}}$$

15. Correct Option: (a)

Given: $y^2 (2 - x) = x^3$

$$\Rightarrow (2-x)2y\frac{dy}{dx} + y^2(-1) = 3x^2$$

$$\Rightarrow (2-x)2y\frac{dy}{dx} = 3x^2 + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2+y^2}{2y(2-x)}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,\,1)} = 2$$

16. Correct Option: (a)

Given: $y = x^2 - 5x + 6$

$$\Rightarrow \frac{dy}{dx} = 2x - 5$$

Slope at (2, 0) is $m_1 = -1$

Slope at (3, 0) is $m_2 = 1$

Angle between the tangents is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{0}\right)$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

17. Correct Option: (a)

Given: $y = x^3 + 6x^2 + 6$

$$\Rightarrow f'(x) = \frac{dy}{dx} = 3x^2 + 12x = 3x(x+4)$$

Taking $f'(x) = 0$, we get, $x = 0$ & $x = -4$

Now, $f'(x) > 0$ for $x < -4$ and $x > 0$.

And, $f'(x) < 0$ for $-4 < x < 0$.

Hence, f(x) increases on the interval $(-\infty, -4) \cup (0, \infty)$.

18. Correct Option: (b)

Let $f(x) = x - x^2$

Therefore, $f'(x) = 1 - 2x$

Taking $f'(x) = 0$, we get, $x = \frac{1}{2}$

Now, $f''(x) = -2$

At $x = \frac{1}{2}$, $f''(x) < 0$

Therefore, f(x) is maximum at $x = \frac{1}{2}$.

19. Correct Option: (a)

Value of Z at the corner points is as follows:

Points	$Z = 11x + 7y$
(0, 3)	Z = 21
(3, 2)	Z = 47
(0, 5)	Z = 35

Therefore, minimum value of Z is 21.

20. Correct Option: (a)

$x + 2y \leq 3$

i.e. $x \leq 3 - 2y$

$3x + 4y \geq 12$

i.e. $x \geq 4 - 4y/3$

Therefore, $4 - 4y/3 \leq x \leq 3 - 2y$

Therefore, $4 - 4y/3 \leq 3 - 2y$

$2y - 4y/3 \leq 3 - 4$

$y \leq -3/2$

Since $y \geq 1$

So, there is no value of y satisfying these equations.

Hence, no solution exist.

SECTION B

21. Correct Option: (c)

A relation R is an equivalence relation if is reflexive, symmetric and transitive.

For reflexive, we must have elements (1, 1), (2, 2) and (3, 3) in R.

It should contain (1, 2) and (2, 1).

So, one of the possible relations is $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

If we add one more ordered pair, say (3, 2), we must have (2, 3) as an element.

As (1, 2) & (2, 3) in that relation, we must have (1, 3) as well as it is transitive.

Also, we should have (3, 1) as it is symmetric.

The another relation is R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (3, 2), (2, 3), (1, 3), (3, 1)}.
Hence, only two equivalence relations are possible.

22. Correct Option: (d)

We have $f(x) = \frac{1}{x}$, for all $x \in R$

But at x = 0, f is not defined.

23. Correct Option: (c)
Here, f is not one-one because f(1) = f(2), 1 ≠ 2
When n is odd, f(n) ∈ {1, 2, 3, …}
When n is even, f(n) ∈ {1, 2, 3, …}
So, the range of f is {1, 2, 3, …} i.e. N.
Thus, f is onto.

24. Correct Option: (b)

$$\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left(\cos\left(2\pi + \frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right)$$

Since, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$$\sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \frac{\pi}{2} - \cos^{-1}\left(\cos\frac{3\pi}{5}\right)$$

$$= \frac{\pi}{2} - \frac{3\pi}{5}$$

$$= -\frac{\pi}{10}$$

25. Correct Option: (c)

Since, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}x > \frac{\pi}{2} - \sin^{-1}x$$

$$\Rightarrow \sin^{-1}x > \frac{\pi}{4}$$

$$\Rightarrow x > \sin\frac{\pi}{4}$$

$$\Rightarrow x > \frac{1}{\sqrt{2}}$$

Hence, x lies in the interval $\left(\frac{1}{\sqrt{2}}, 1\right)$.

26. Correct Option: (b)

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

As A + A′ = I

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, $\alpha = \frac{\pi}{3}$.

27. Correct Option: (d)

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = (y+k)\left[(y+k)^2 - y^2\right] - y\left[y(y+k) - y^2\right] + y\left[y^2 - y(y+k)\right]$$

$$= (y+k)^3 - y^2(y+k) - y^2(y+k) + y^3 + y^3 - y^2(y+k)$$

$$= y^3 + k^3 + 3y^2k + 3yk^2 + 2y^3 - 3y^3 - 3y^2k$$

$$= k^2(k+3y)$$

28. Correct Option: (d)

Given: adj B = A

Therefore, det(adj B) = det(A)

$$\Rightarrow (\det B)^2 = \det A = -6 + 2(-12) + 4(4+4)$$

$$\Rightarrow (\det B)^2 = 2$$

$$\Rightarrow \det B = \pm\sqrt{2}$$

$$\Rightarrow B^{-1} = \frac{1}{\det B}(\text{adj}B) = \pm\frac{1}{\sqrt{2}}A$$

29. Correct Option: (d)

$$A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1-1+1 & -1+1-1 & 1-1+1 \\ 1-1+1 & -1+1-1 & 1-1+1 \\ 1-1+1 & -1+1-1 & 1-1+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = A$$

A3 = $A^2A = A^2 = A$

Similarly, $A^4 = A$ and $A^5 = A$

Therefore, $A^5 - A^4 - A^3 + A^2 = A - A - A + A = O$.

30. Correct Option: (b)

Given: $y = e^{-x}$

$$\Rightarrow \frac{dy}{dx} = -e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = y$$

31. Correct Option: (b)

Given: $x = t^2 + 1$, $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2t \;\;\&\;\; \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{a}{t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{a}{t^2} \times \frac{dt}{dx}$$

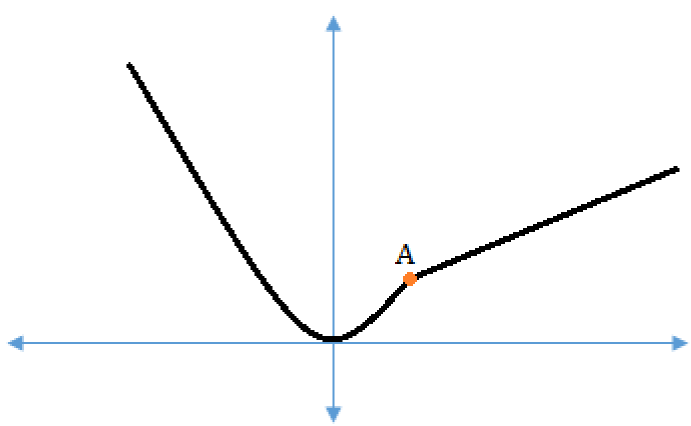
$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{a}{2t^3}$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=a} = -\frac{1}{2a^2}$$

32. Correct Option: (a)

Given: $f(x) = \begin{cases} x^2 & , \text{ for } x < 1 \\ 2 - x, & \text{ for } x \geq 1 \end{cases}$

Graph of this function is



From the graph, At the point A(1, 1), we can't draw a tangent.

Hence, f(x) is not differentiable at x = 1.

33. Correct Option: (a)

Given curve is $x^2 - xy + y^2 = 27$ ... (i)

$$\Rightarrow 2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2y - x) = y - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Slope of a line parallel to x-axis is 0.

$$\Rightarrow \frac{y - 2x}{2y - x} = 0$$

$$\Rightarrow y = 2x \quad \dots (ii)$$

Using this in (i), we get

$x^2 - 2x^2 + 4x^2 = 27$

Therefore, $x^2 = 9$

Thus, $x = \pm 3$

So, $y = \pm 6$

Hence, the points are (3, 6) and (−3, −6).

34. Correct Option: (b)

Length of the wire = 20 cm

Therefore, Length of wire = 2Radius + Arc length

i.e. $2r + l = 20$ cm

Therefore, $l = 2(10 - r) = \frac{1}{2}\pi r^2$

Area of sector (A) $= \frac{\theta}{360} \times \pi r^2$

As $\theta = \frac{l}{r}$

$$\Rightarrow A = \frac{l}{360} \times \pi r = \frac{20 - 2r}{360} \times \pi r = \frac{10\pi r - \pi r^2}{180}$$

$$\Rightarrow \frac{dA}{dr} = \frac{10\pi - 2\pi r}{180} = \frac{5\pi - \pi r}{90}$$

Taking $\frac{dA}{dr} = 0$, we get, $r = 5$

Therefore, $l = 20 - 10 = 10$

Length of an arc $= \frac{\theta}{360} \times 2\pi r$

$\Rightarrow 10 = \frac{\theta}{360} \times 10\pi$

$\Rightarrow \theta = \frac{360}{\pi}$

$\Rightarrow A = \frac{360}{\pi \times 360} \times \pi \times 5^2$

$\Rightarrow A = 25$ sq cm

35. Correct Option: (d)

Let f(x) = x − sin x

Therefore, f′(x) = 1 − cos x

Now, f(x) decreases when f′(x) < 0

i.e. 1 < cos x

But, −1 ≤ cos x ≤ 1

Thus, there is no such value of x.

36. Correct Option: (a)

Let $y^2 = 4ax$ … (i)

$\Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{2a}{y}$

And, $ay = 2x^2$ … (ii)

$\Rightarrow a\frac{dy}{dx} = 4x$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{4x}{a}$

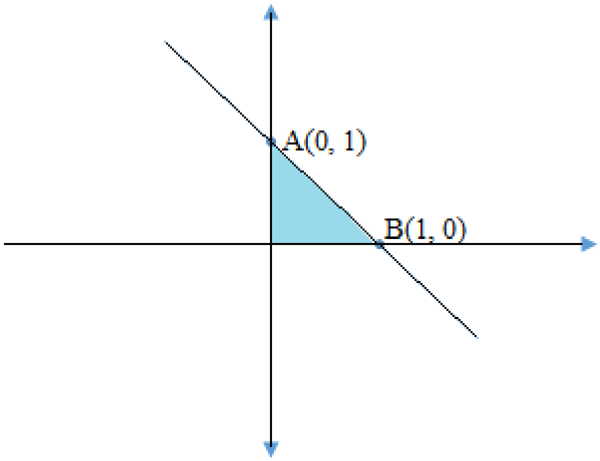
Angle between the curves is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{4x}{a} - \frac{2a}{y}}{1 + \frac{4x}{a}\left(\frac{2a}{y}\right)}\right| = \left|\frac{\frac{4x}{a} - \frac{2a}{y}}{1 + \frac{8x}{y}}\right|$$

$$\Rightarrow \tan\theta\Big|_{(a,\ 2a)} = \left|\frac{\frac{4a}{a} - \frac{2a}{2a}}{1 + \frac{8a}{2a}}\right| = \left|\frac{4 - 1}{1 + 4}\right| = \frac{3}{5}$$

37. Correct Option: (b)

Graph of $x + y \leq 1$ is given by



The corner points are O(0, 0), A(0, 1) and B(1, 0)

So, the maximum value of Z is 4.

38. Correct Option: (b)

The first line intersects coordinate axes at (0, 104) and (52, 0)

Its equation is $2x + y = 104$

So, the inequality becomes $2x + y \leq 104$

The second line intersects coordinate axes at (0, 38) and (76, 0)

Its equation is $x + 2y = 76$

So, the inequality becomes $x + 2y \leq 76$

39. Correct Option: (b)

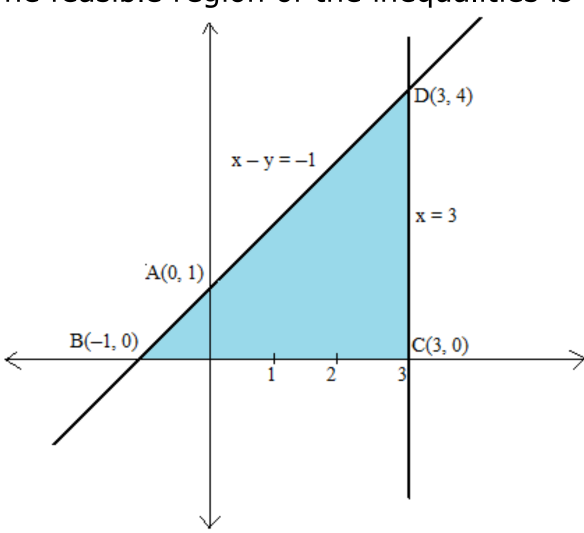
As the minimum value occurs at (3, 4) and (4, 3)

Therefore, $3a + 4b = Z$ & $4a + 3b = Z$

Therefore, $a = b$.

40. Correct Option: (c)

The feasible region of the inequalities is given by



Corner Points	Value of Z = 3x + 4y
A(0, 1)	Z = 4
B(−1, 0)	Z = −3
C(3, 0)	Z = 9
D(3, 4)	Z = 25

Thus, the maximum value of Z is 25.

SECTION C

41. Correct Option: (d)

We have, $a^2 - 7aa + 6a^2 = 7a^2 - 7a^2 = 0$

Therefore, aRa for all a in Z.

So, R is reflexive.

Let aRb, then $a^2 - 7ab + 6b^2 = 0$

Consider, $b^2 - 7ba + 6a^2 = b^2 - (a^2 + 6b^2) + 6a^2 = 5a^2 - 5b^2$

So, f is not symmetric.

42. Correct Option: (d)

$$\begin{vmatrix} \underline{|1} & \underline{|2} & \underline{|3} \\ 2\underline{|2} & 3\underline{|3} & 4\underline{|4} \\ \underline{|3} & \underline{|4} & \underline{|5} \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 4 & 18 & 96 \\ 6 & 24 & 120 \end{vmatrix}$$

$$= 2(6)\begin{vmatrix} 1 & 2 & 6 \\ 2 & 9 & 48 \\ 1 & 4 & 20 \end{vmatrix}$$

$$= 12\left[180 - 192 - 2(40 - 48) + 6(8 - 9)\right]$$

$$= 12\left[-12 + 16 - 6\right]$$

$$= -24$$

43. Correct Option: (b)

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1 + \tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \cos^2\theta\begin{bmatrix} 1 - \tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1 - \tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \cos^2\theta\left(1 - \tan^2\theta\right) = a \quad \& \quad 2\cos^2\theta\tan\theta = b$$

$$\Rightarrow a = \cos 2\theta \quad \& \quad b = \sin 2\theta$$

44. Correct Option: (a)

The curve is $3y = 6x - 5x^3$

$$\Rightarrow \frac{dy}{dx} = 2 - 5x^2$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\left(1, \frac{1}{3}\right)} = -3$$

So, the slope of normal will be $\frac{1}{3}$.

Equation of normal at $\left(1, \ \frac{1}{3}\right)$ is given by

$$y - \frac{1}{3} = \frac{1}{3}(x - 1)$$

$$\Rightarrow x - 3y = 0$$

The point (3, 1) satisfies this equation.

Hence, the normal passes through (3, 1).

45. Correct Option: (c)

Given: $y = \sin(2 \sin^{-1}x)$

$$\Rightarrow y_1 = \cos\left(2\sin^{-1}x\right)\frac{d}{dx}\left(2\sin^{-1}x\right)$$

$$\Rightarrow y_1 = \frac{2\cos\left(2\sin^{-1}x\right)}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1\sqrt{1-x^2} = 2\cos\left(2\sin^{-1}x\right)$$

$$\Rightarrow y_1\left(\frac{-2x}{2\sqrt{1-x^2}}\right) + \sqrt{1-x^2}\left(y_2\right) = -2\sin\left(2\sin^{-1}x\right)\left(\frac{2}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow y_1\left(\frac{-x}{\sqrt{1-x^2}}\right) + \sqrt{1-x^2}\left(y_2\right) = -\frac{4y}{\sqrt{1-x^2}}$$

$$\Rightarrow -xy_1 + y_2\left(1-x^2\right) = -4y$$

$$\Rightarrow y_2\left(1-x^2\right) = xy_1 - 4y$$

46. Correct Option: (c)

Volume of half cylinder (V) = $\frac{1}{2}\pi r^2 h$

47. Correct Option: (b)

Total surface area of the half cylinder (S) = $\pi rh + \pi r^2 + 2rh$

48. Correct Option: (c)

$$S = \pi rh + \pi r^2 + 2rh$$

$$= rh(\pi + 2) + \pi r^2$$

$$= \pi r^2 + \frac{2V(\pi + 2)}{\pi r}$$

49. Correct Option: (a)

$$S = \pi r^2 + \frac{2V(\pi + 2)}{\pi r}$$

$$\Rightarrow \frac{dS}{dr} = 2\pi r - \frac{2V(\pi + 2)}{\pi r^2}$$

Taking $\frac{dS}{dr} = 0,$ we get

$$\pi^2 r^3 = V(\pi + 2)$$

50. Correct Option: (d)

The total surface area is minimum when $\pi^2 r^3 = V(\pi + 2)$

Since, $V = \frac{1}{2}\pi r^2 h$

$$\Rightarrow \frac{\pi^2 r^3}{(\pi + 2)} = \frac{1}{2}\pi r^2 h$$

$$\Rightarrow \frac{\pi r}{(\pi + 2)} = \frac{1}{2}h$$

$$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$$