TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

(1) 5 (2) 17 (3) 12 (4) 7

Ans. **(4)**

Sol. Normal vector of plane

$$= \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix} = -4\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

equation of plane is $5(x-7) + 2y - 3(z - 6) = 0$

$5x + 2y - 3z = 17$

2. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :-

(1) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(2) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Ans. **(1)**

Sol. $D = (1 + \sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta = (1 - \sin\theta\cos\theta)^2$

$\Rightarrow$ roots are $\beta = \operatorname{cosec}\theta$ and $\alpha = \cos\theta$

$$\Rightarrow \sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{n}(-\sin\theta)^n$$

$$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$$

3. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$ is not differentiable. Then the set K is equal to :-

(1) $\{\pi\}$ (2) $\{0\}$

(3) ϕ (an empty set) (4) $\{0, \pi\}$

Ans. **(3)**

Sol. $f(x) = \sin|x| - |x| + 2(x - \pi)\cos x$

$\because \sin|x| - |x|$ is differentiable function at $x=0$

$\therefore k = \phi$

4. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

Ans. **(2)**

Sol. $\frac{2b^2}{a} = 8$ and $2ae = 2b$

$\Rightarrow \frac{b}{a} = e$ and $1 - e^2 = e^2 \Rightarrow e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = 4\sqrt{2}$ and $a = 8$

so equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{32} = 1$

5. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :-

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

Ans. **(4)**

Sol. Vertex is $(a^2, 0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.\left(a^2\right) = 250$

$\Rightarrow a^3 = 125$ or $a = 5$

6. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals :-

(1) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{10}$

(4) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

Ans. **(1)**

Sol. $I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$

$I = \frac{1}{2}\int_{\pi/6}^{\pi/4} \frac{\tan^4 x \sec^2 x\, dx}{\left(1 + \tan^{10} x\right)}$ Put $\tan^5 x = t$

$I = \frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{dt}{1+t^2} = \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\frac{1}{9\sqrt{3}}\right)$

7. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{50} x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:-

(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25

Ans. **(4)**

Sol. $(10 + x)^{50} + (10 - x)^{50}$

$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48}$, $a_0 = 2.10^{50}$

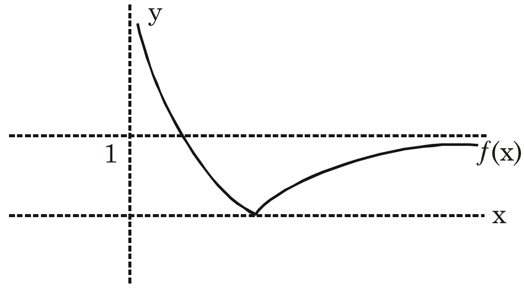
$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$

8. Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is :-

(1) Injective only
(2) Not injective but it is surjective
(3) Both injective as well as surjective
(4) Neither injective nor surjective

Ans. **(Bonus)**

Sol. $f(x) = \left|1 - \frac{1}{x}\right| = \frac{|x-1|}{x} = \begin{cases} \frac{1-x}{x} & 0 < x \le 1 \\ \frac{x-1}{x} & x \ge 1 \end{cases}$



$\Rightarrow f(x)$ is not injective

but range of function is $[0,\infty)$

Remark : If co-domain is $[0,\infty)$, then $f(x)$ will be surjective

9. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :-

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

Ans. **(2)**

Sol. 7,
1,6
2,5
3,4
1,2,4

$P = \frac{5}{2^{20}}$

10. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

(1) (2, 4, 7) (2) (–2, 4, 7)
(3) (2, –4, –7) (4) (2, –4, 7)

Ans. **(3)**

Sol. Point on L_1 $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\Rightarrow \lambda + 3 = 7\mu - 5$ ...(i)

$3\lambda - 1 = -6\mu + 2$ ...(ii) $\Rightarrow \lambda = -1, \mu = 1$

point R(2,–4,7)

Reflection is (2,–4,–7)

11. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :-

(1) (15)! × 6! (2) $5^6 \times 15$

(3) 5! × 6! (4) $6^5 \times (15)!$

Ans. (1)

Sol. $f(k) = 3m\,(3,6,9,12,15,18)$

for $k = 4,8,12,16,20$ 6.5.4.3.2 ways

For rest numbers 15! ways

Total ways = 6!(15!)

12. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

Ans. (1)

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

13. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when y(1) = 1, is :-

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

Ans. (4)

Sol. $x - y = t \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$$\Rightarrow 1 - \frac{dt}{dx} = t^2 \Rightarrow \int \frac{dt}{1-t^2} = \int 1dx$$

$$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right) = x + \lambda$$

$$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right) = x + \lambda \quad \text{given } y(1) = 1$$

$$\Rightarrow \frac{1}{2}\ell n(1) = 1 + \lambda \Rightarrow \lambda = -1$$

$$\Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right) = 2(x-1)$$

$$\Rightarrow -\ell n\left(\frac{1-x+y}{1+x-y}\right) = 2(x-1)$$

14. Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :-

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

Ans. (2)

Sol. $|A|^2.|B| = 8$ and $\frac{|A|}{|B|} = 8 \Rightarrow |A| = 4$ and $|B| = \frac{1}{2}$

$\therefore \det(BA^{-1}.B^T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

15. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to :-

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

Ans. (1)

Sol. $\sqrt{2x-1} = t \Rightarrow 2x - 1 = t^2 \Rightarrow 2dx = 2t.dt$

$$\int \frac{x+1}{\sqrt{2x-1}}dx = \int \frac{\frac{t^2+1}{2}+1}{t}t\,dt = \int \frac{t^2+3}{2}dt$$

$$= \frac{1}{2}\left(\frac{t^3}{3} + 3t\right) = \frac{t}{6}\left(t^2+9\right) + c$$

$$= \sqrt{2x-1}\left(\frac{2x-1+9}{6}\right) + c = \sqrt{2x-1}\left(\frac{x+4}{3}\right) + c$$

$$\Rightarrow f(x) = \frac{x+4}{3}$$

16. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\dfrac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to :-

(1) 4 (2) $\dfrac{4\sqrt{3}}{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

Ans. **(3)**

Sol. p (probability of getting white ball) = $\dfrac{30}{40}$

$q=\dfrac{1}{4}$ and n = 16

mean =np = $16.\dfrac{3}{4}=12$

and standard diviation

$=\sqrt{npq}=\sqrt{16.\dfrac{3}{4}.\dfrac{1}{4}}=\sqrt{3}$

17. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:-

(1) $5x + 3y - 11 = 0$ (2) $3x - 5y + 7 = 0$
(3) $3x + 5y - 13 = 0$ (4) $5x - 3y + 1 = 0$

Ans. **(4)**

Sol. co-ordinates of point D are (4,7)
$\Rightarrow$ line AD is $5x - 3y + 1 = 0$

18. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :-

(1) 2 (2) $\dfrac{13}{6}$ (3) $\dfrac{13}{8}$ (4) $\dfrac{13}{12}$

Ans. **(4)**

Sol. 2b = 5 and 2ae = 13

$b^2 = a^2(e^2 - 1) \Rightarrow \dfrac{25}{4}=\dfrac{169}{4}-a^2$

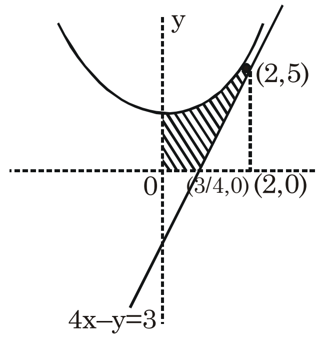
$\Rightarrow a=6 \Rightarrow e=\dfrac{13}{12}$

19. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :-

(1) $\dfrac{14}{3}$ (2) $\dfrac{187}{24}$ (3) $\dfrac{37}{24}$ (4) $\dfrac{8}{3}$

Ans. **(3)**

Sol.



Area $=\int_0^2\left(x^2+1\right)dx-\dfrac{1}{2}\left(\dfrac{5}{4}\right)(5)=\dfrac{37}{24}$

20. Let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\dfrac{3}{\sqrt{2}}$, then the sum of all possible values of β is :-

(1) 2 (2) 1 (3) 3 (4) 4

Ans. **(2)**

Sol. Angle bisector is $x - y = 0$

$\Rightarrow \dfrac{|\beta-(1-\beta)|}{\sqrt{2}}=\dfrac{3}{\sqrt{2}}$

$\Rightarrow |2\beta-1|=3$

$\Rightarrow \beta=2 \text{ or } -1$

21. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $= (a + b + c)(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to :-

(1) $-(a + b + c)$ (2) $2(a + b + c)$
(3) abc (4) $-2(a + b + c)$

Ans. **(4)**

Sol. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 2c & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$

$\Rightarrow x = -2(a + b + c)$

22. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + ... + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then α is equal to :-

(1) 2^{100} (2) 200 (3) 2^{99} (4) 202

Ans. **(1)**

Sol. ${}^{101}C_1 + {}^{101}C_2S_1 + ... + {}^{101}C_{101}S_{100}$
$= \alpha T_{100}$

${}^{101}C_1 + {}^{101}C_2(1 + q) + {}^{101}C_3(1 + q + q^2) + ... + {}^{101}C_{101}(1 + q + ... + q^{100})$

$$= 2\alpha \frac{\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$$

$\Rightarrow {}^{101}C_1(1 - q) + {}^{101}C_2(1 - q^2) + ... + {}^{101}C_{101}(1 - q^{101})$

$$= 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1)$

$$= 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1 - \left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow \alpha = 2^{100}$

23. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :-

(1) A hyperbola (2) A parabola
(3) A straight line (4) An ellipse

Ans. **(2)**

Sol. Let equation of circle is

$x^2 + y^2 + 2fx + 2fy + e = 0$, it passes through (0, 2b)

$\Rightarrow 0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$\Rightarrow 4b^2 + 4f + c = 0$ ...(i)

$2\sqrt{g^2 - c} = 4a$ ...(ii)

$g^2 - c = 4a^2 \Rightarrow c = \left(g^2 - 4a^2\right)$

Putting in equation (1)

$\Rightarrow 4b^2 + 4f + g^2 - 4a^2 = 0$

$\Rightarrow x^2 + 4y + 4(b^2 - a^2) = 0$, it represent a parabola.

24. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :-

(1) 3 : 1 (2) 4 : 1 (3) 2 : 1 (4) 1 : 3

Ans. **(1)**

Sol. $a + 18d = 0$ ...(1)

$$\frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d} = \frac{3}{1}$$

25. Let $f(x) = \dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$, where a, b and d are non-zero real constants. Then :-

(1) f is a decreasing function of x
(2) f is neither increasing nor decreasing function of x
(3) f' is not a continuous function of x
(4) f is an increasing function of x

Ans. **(4)**

Sol.
$$f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$f'(x) = \frac{a^2}{\left(a^2+x^2\right)^{3/2}} + \frac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}} > 0 \, \forall x \in R$$

$f(x)$ is an increasing function.

26. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to :-

(1) $\frac{5}{4}$ (2) $\frac{\sqrt{41}}{4}$ (3) $\frac{\sqrt{34}}{3}$ (4) $\frac{5}{3}$

Ans. **(4)**

Sol. $|z| + z = 3 + i$

$z = 3 - |z| + i$

Let $3 - |z| = a \Rightarrow |z| = (3 - a)$

$\Rightarrow z = a + i \Rightarrow |z| = \sqrt{a^2+1}$

$\Rightarrow 9 + a^2 - 6a = a^2 + 1 \Rightarrow a = \frac{8}{6} = \frac{4}{3}$

$\Rightarrow |z| = 3 - \frac{4}{3} = \frac{5}{3}$

27. All x satisfying the inequality $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, lie in the interval:-

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) $(\cot 5, \cot 4)$

(3) $(\cot 2, \infty)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

Ans. (4)

Sol. $\cot^{-1}x > 5, \quad \cot^{-1}x < 2$

$\Rightarrow x < \cot 5,\ x > \cot 2$

28. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :-

(1) (3, 4, 5) (2) (19, 7, 25)

(3) (7, 19, 25) (4) (5, 12, 13)

28. Ans. (3)

Sol. $b + c = 11\lambda,\ c + a = 12\lambda,\ a + b = 13\lambda$

$\Rightarrow a = 7\lambda,\ b = 6\lambda,\ c = 5\lambda$

(using cosine formula)

$$\cos A = \frac{1}{5}, \cos B = \frac{19}{35}, \cos C = \frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

29. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

Ans. (2)

Sol. $$\frac{x^m y^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)} = \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)} \le \frac{1}{4}$$

using AM $\geq$ GM

30. $\lim\limits_{x\to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :-

(1) 2 (2) 0 (3) 4 (4) 1

Sol. $$\lim_{x\to 0} \frac{x \tan^2 2x}{\tan 4x \sin^2 x} = \lim_{x\to 0} \frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$