JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. A sphere of 10 cm radius has a uniform thickness of ice around it. If the ice is melting at the rate of 50 cm^3/min when thickness is 5 cm, then the rate of change of thickness is

a. $\frac{1}{12\pi}$ b. $\frac{1}{18\pi}$
c. $\frac{1}{9\pi}$ d. $\frac{1}{36\pi}$

Answer: (b)

Solution:

Let thickness of ice be x cm.

Therefore, net radius of sphere $= (10 + x)$ cm

Volume of sphere $V = \frac{4}{3}\pi(10 + x)^3$

$\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$

At $x = 5, \frac{dV}{dt} = 50\ \text{cm}^3/\text{min}$

$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi}\ \text{cm/min}$

2. The number of real roots of $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

a. 1 b. 2
c. 3 d. 4

Answer: (a)

Solution:

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

Let $e^x + \frac{1}{e^x} = u$

Then, $u^2 + u - 6 = 0$

$\Rightarrow u = 2, -3$

$u \neq -3$ as $u > 0$ ($\because e^x > 0$)

$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

Hence, only one real solution is possible.

3. If $f'(x) = \tan^{-1}(\sec x + \tan x), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$, then the value of $f(1)$ is

a. $\frac{\pi-1}{4}$

b. $\frac{\pi+1}{4}$

c. $\frac{\pi+1}{2}$

d. 0

Answer: (b)

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}$$

4. The number of solutions of $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0, 2\pi]$ is

a. 2

b. 4

c. 8

d. 6

Answer: (c)

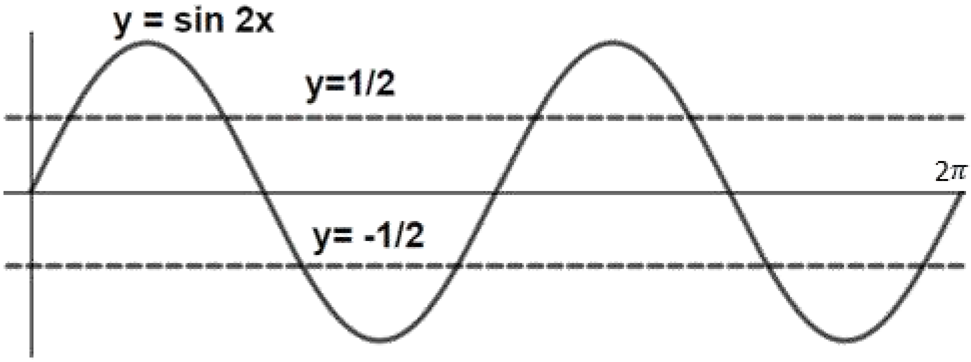
Solution:

$\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0,2\pi]$

$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x \cos x| = \frac{1}{4}$

$\therefore \sin 2x = \pm\frac{1}{2}$



$\therefore$ We have 8 solutions for $x \in [0,2\pi]$

5. If $e_1$and $e_2$are the eccentricities of $\frac{x^2}{18}+\frac{y^2}{4}=1$ and $\frac{x^2}{9}-\frac{y^2}{4}=1$, respectively. If the points (e_1, e_2) lies on the ellipse $15x^2+3y^2=k$. Then the value of k is

a. 16
b. 14
c. 15
d. 17

Answer: (a)

Solution:

$$e_1 = \sqrt{1-\frac{4}{18}} = \frac{\sqrt{7}}{3} \;\&\; e_2 = \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$\because (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$\therefore 15e_1^2 + 3e_2^2 = k$

$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$

6. The value of $\int \frac{dx}{(x-3)^{\frac{6}{7}}\times(x+4)^{\frac{8}{7}}}$ is –

a. $7\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+c$

b. $7\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}}+c$

c. $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+c$

d. $7\left(\frac{x+4}{x-3}\right)^{\frac{6}{7}}+c$

Answer: (c)

Solution:

$I=\int \frac{dx}{(x-3)^{\frac{6}{7}}\times(x+4)^{\frac{8}{7}}}$

$\Rightarrow I\ =\int \frac{(x+4)^{\frac{6}{7}}dx}{(x-3)^{\frac{6}{7}}\times(x+4)^{2}}=\int \left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}}\times\frac{dx}{(x+4)^{2}}$

Put $\frac{x-3}{x+4}=t\Rightarrow dt=7\left(\frac{1}{(x+4)^{2}}\right)dx$

$\Rightarrow I=\frac{\int t^{-\frac{6}{7}}}{7}dt=t^{\frac{1}{7}}+c=\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+c$

7. If $\left|\frac{z-i}{z+2i}\right|=1$, $|z|=\frac{5}{2}$ then the value of $|z+3i|$ is

a. $\sqrt{10}$

b. $\sqrt{5}$

c. $\frac{7}{2}$

d. $\sqrt{3}$

Answer: (c)

Solution:

If $\left|\frac{z-i}{z+2i}\right|=1$ & $|z|=\frac{5}{2}$

$\Rightarrow |z-i|=|z+2i|$

$\Rightarrow x^2+(y-1)^2=x^2+(y+2)^2$

$\Rightarrow y-1=\pm(y+2)$

$\Rightarrow y-1=-y-2$

$\Rightarrow y=-\frac{1}{2}$

$\Rightarrow |z|=\frac{5}{2}\Rightarrow x^2+y^2=\frac{25}{4}$

$\Rightarrow x^2+\frac{1}{4}=\frac{25}{4}$

$\Rightarrow x=\pm\sqrt{6}$

$|z+3i| = \sqrt{x^2+(y+3)^2}$

$\Rightarrow |z+3i| = \frac{7}{2}$

8. The value of $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \ldots \infty$ is
   a. 2
   b. 1
   c. $\sqrt{2}$
   d. $2^{\frac{1}{4}}$

   Answer: (c)

   Solution:

   $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \ldots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \ldots \infty$

   $\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \ldots \infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\infty}$

   $\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)} = \sqrt{2}$

9. The value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is –
   a. $\frac{1}{2}$
   b. $-\frac{1}{2}$
   c. $\frac{1}{\sqrt{2}}$
   d. $\frac{1}{2\sqrt{2}}$

   Answer: (d)

   Solution:

   $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8} = \cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

   $= 4\left[\cos^6\frac{\pi}{8} - \sin^6\frac{\pi}{8}\right] + 3\left[\sin^4\frac{\pi}{8} - \cos^4\frac{\pi}{8}\right]$

   $= 4\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right] - 3\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]$

   $= \left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[4\left(1 - \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right) - 3\right]$

   $= \cos\frac{\pi}{4}\left[1 - \sin^2\frac{\pi}{4}\right] = \frac{1}{2\sqrt{2}}$

10. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$ is
    a. $4\pi^2$
    b. $2\pi^2$
    c. π^2
    d. $3\pi^2$

    Answer: (c)

Solution:

Let $I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (1)

$I = \int_0^{2\pi} \frac{(2\pi - x)\sin^8(2\pi - x)}{\sin^8(2\pi - x) + \cos^8(2\pi - x)} dx$

$$= \int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x} dx \quad ... (2)$$

Adding (1) & (2) , we get:

$\Rightarrow 2I = 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (3)

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8\left(\frac{\pi}{2} - x\right)}{\sin^8\left(\frac{\pi}{2} - x\right) + \cos^8\left(\frac{\pi}{2} - x\right)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$ ... (4)

Adding (3) & (4) , we get –

$$I = 2\pi \int_0^{\frac{\pi}{2}} 1\, dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

11. If $f(x) = a + bx + cx^2$ where $a, b, c \in \mathbf{R}$ then the value of $\int_0^1 f(x)dx$ is –

a. $\frac{1}{6}\left(f(1) + f(0) - 4f\left(\frac{1}{2}\right)\right)$

b. $\frac{1}{3}\left(f(1) + f(0) + 2f\left(\frac{1}{2}\right)\right)$

c. $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

d. $\frac{1}{6}\left(f(1) - f(0) - 4f\left(\frac{1}{2}\right)\right)$

Answer: (c)

Solution:

$f(x) = a + bx + cx^2$

$f(0) = a, f(1) = a + b + c$

$f\left(\frac{1}{2}\right) = \frac{c}{4} + \frac{b}{2} + a$

$\int_0^1 f(x)dx = \int_0^1 (a + bx + cx^2)dx = a + \frac{b}{2} + \frac{c}{3}$

$= \frac{1}{6}(6a + 3b + 2c) = \frac{1}{6}\left(a + (a + b + c) + (4a + 2b + c)\right)$

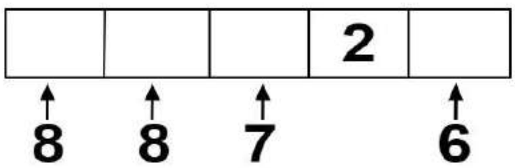
$= \frac{1}{6}\left(f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right)$

12. If the number of ways of forming 5 digit numbers (without repeating any digit), such that the tenth place of the number must be occupied by 2 is $336k$, then the value of k is

a. 5
b. 6
c. 7
d. 8

Answer: (d)

Solution:



Total numbers that can be formed are

$= 8 \times 8 \times 7 \times 6$

$= 8 \times 336$

$\therefore k = 8$

13. If D is the centroid of the ΔABC having vertices $A(3, -1), B(1, 3), C(2, 4)$ and P is the point if intersection of lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, then which of the following point lies on the line joining D and P?

a. $(-9, -6)$
b. $(9, -6)$
c. $(9, 6)$
d. $(-9, -7)$

Answer: (a)

Solution:

Coordinates of D are $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2)$

Point of intersection of two lines

$x + 3y - 1 = 0$ and $3x - y + 1 = 0$

is $P\left(\frac{-1}{5}, \frac{2}{5}\right)$

Equation of line DP is $8x - 11y + 6 = 0$

Point $(-9, -6)$ lies on DP

14. If $f(x)$ is twice differentiable and continuous function in $x \in [a, b]$. Also $f'(x) > 0$ and $f''(x) < 0$ and $c \in (a, b)$, then $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

a. 1

b. $\frac{a+b}{b-c}$

c. $\frac{b-c}{c-a}$

d. $\frac{c-a}{b-c}$

Answer: (d)

Solution:

$\therefore c \in (a, b)$ and f is twice differentiable and continuous function (a, b)

$\therefore$ LMVT is applicable

For $p \in (a, c)$, $\quad f'(p) = \frac{f(c)-f(a)}{c-a}$

For $q \in (c, b)$, $\quad f'(q) = \frac{f(b)-f(c)}{b-c}$

$\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(p) > f'(q)$

$$\Rightarrow \frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$$

$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ (as $f'(x) > 0 \Rightarrow f(x)$ is increasing)

15. If three planes
$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$
intersect in a line, then $\alpha + \beta =$

a. -10

b. 0

c. 2

d. 10

Answer: (d)

Solution:

The given planes intersect in a line

$\therefore D = D_x = D_y = D_z = 0$

$D = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$$

$\Rightarrow \alpha = -3$

$D_z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

16. $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If mean and variance of observations $(x_1 - 3), (x_2 - 3) \ldots\ldots (x_{10} - 3)$ is λ and μ respectively, then ordered pair (λ, μ) is

a. (1,1)
b. (1,3)
c. (3,1)
d. (3,3)

Answer: (d)

Solution:

$$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 60$$

$$\lambda = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\mu = Var(x_i - 3) = Var(x_i - 5) = \frac{\sum_{i=1}^{10}(x_i - 5)^2}{10} - \left(\frac{\sum(x_i - 5)}{10}\right)^2$$

$$= \frac{40}{10} - \left(\frac{10}{10}\right)^2 = 3$$

17. 20 cards are placed in a bag with 10 named as A and another 10 named as B. If cards are draw n one by one (with replacement), then the probability that second A comes before third B is

a. $\frac{11}{16}$
b. $\frac{7}{16}$
c. $\frac{9}{16}$
d. $\frac{13}{16}$

Answer: (a)

Solution:

Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible → $AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

18. The negation of '$\sqrt{5}$ is an integer or 5 is an irrational number' is
    a. $\sqrt{5}$ is an integer and 5 is not an irrational number.
    b. $\sqrt{5}$ is not an integer and 5 is not an irrational number.
    c. $\sqrt{5}$ is not an integer or 5 is not an irrational number.
    d. $\sqrt{5}$ is not an integer and 5 is an irrational number.

Answer: (b)

Solution:

p: $\sqrt{5}$ is an integer

q: 5 is an irrational number

Given statement : $p \vee q$

Required negation statement: $\sim(p \vee q) = \sim p \wedge \sim q$

'$\sqrt{5}$ is not an integer and 5 is not an irrational number'

19. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}(A)$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is
    a. 2
    b. 4
    c. 8
    d. 16

Answer: (c)

Solution:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$$

$B = \text{adj}(A) \Rightarrow |\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^4 = 6^4$

$|C| = |3A| = 3^3|A| = 3^3 \times 6$

$$\frac{|\text{adj } B|}{|C|} = \frac{6^4}{3^3 \times 6} = \frac{2^3 \times 3^3}{3^3} = 8$$

20. If a circle touches y-axis at (0,4) and passes through (2,0), then which of the following can be the tangent to the circle?

a. $3x + 4y - 24 = 0$

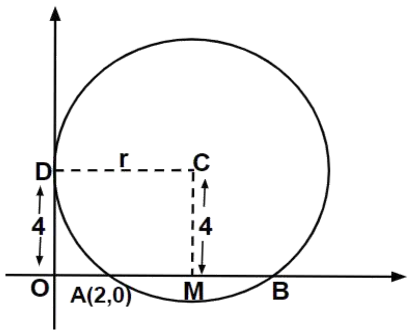
b. $4x - 3y - 17 = 0$

c. $4x + 3y - 6 = 0$

d. $3x + 4y - 6 = 0$

Answer: (d)

Solution:



$OD^2 = OA \times OB \Rightarrow 16 = 2 \times OB \Rightarrow OB = 8$

$\therefore AB = 6$

$\therefore AM = 3, CM = 4 \Rightarrow CA = 5$

$\therefore OM = 5$

Centre will be (5,4) and radius is 5

Now checking option (d)

$3x + 4y - 6 = 0$

$$\frac{15 + 16 - 6}{\sqrt{3^2 + 4^2}} = 5 \ \ (p = r)$$

21. $(1 + x)\frac{dy}{dx} = [(1 + x)^2 + (y - 3)]$. If $y(2) = 0$, then the value of $y(3)$ is

Answer: (3)

Solution:

$$(1 + x)\frac{dy}{dx} = [(1 + x)^2 + (y - 3)]$$

$$\Rightarrow (1 + x)\frac{dy}{dx} - y = (1 + x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1 + x)}y = 1 + x - \frac{3}{1 + x}$$

$$\text{I.F.} = e^{-\int \frac{1}{1+x}dx} = \frac{1}{1 + x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2}\, dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At $x = 2, y = 0$, we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$\Rightarrow$ At $x = 3$,

$$y = x^2 - 2x = 9 - 6 = 3$$

$$\Rightarrow y(3) = 3$$

22. Function $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}, & x < 0 \\ b & , x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}} & , x > 0 \end{cases}$ is continuous at $x = 0$. The value of $a + 2b$ is

Answer: (0)

Solution:

$f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \to 0^-} f(x) = b = \lim_{x \to 0^+} f(x)$$

$$b = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{h^{\frac{1}{3}}\left[(1+3h)^{\frac{1}{3}} - 1\right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{1}{3}(1+3h)^{-\frac{2}{3}} \times 3$$

or, $b = 1$

$\lim_{x\to 0^-} f(x) = 1 \Rightarrow \lim_{h\to 0} \frac{\sin(a+2)(-h)) + \sin(-h)}{-h} = 1$

$\Rightarrow a + 3 = 1 \Rightarrow a = -2$

$\Rightarrow a + 2b = 0$

23. The coefficient of x^4 in $(1 + x + x^2)^{10}$ is

Answer: (615)

Solution:

General term of the given expression is given by $\frac{10!}{p!q!r!} x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient $= \frac{10!}{6!\times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient $= \frac{10!}{7!\times 2!\times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient $= \frac{10!}{8!\times 2!} = 45$

Therefore, sum $= 615$

24. If $\vec{P} = (a+1)\hat{\imath} + a\hat{\jmath} + a\hat{k}$
$\vec{Q} = a\hat{\imath} + (a+1)\hat{\jmath} + a\hat{k}$
$\vec{R} = a\hat{\imath} + a\hat{\jmath} + (a+1)\hat{k}$
and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P}.\vec{Q})^2 - \lambda|\vec{R}\times\vec{Q}|^2 = 0$, then value of λ is

Answer: (1)

Solution:

As $\vec{P}, \vec{Q}, \vec{R}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$3a + 1 = 0$

$\Rightarrow a = -\frac{1}{3}$

$\vec{P} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} - \hat{k}),\qquad \vec{Q} = \frac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k}),\qquad \vec{R} = \frac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$$\vec{R} \times \vec{Q} = \frac{1}{9}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}) = -\frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$

$\vec{P}.\vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$3(\vec{P}.\vec{Q})^2 - \lambda|\vec{R} \times \vec{Q}|^2 = 0$

$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$

25. Points $A(2,4,0), B(3,1,8), C(3,1,-3), D(7,-3,4)$ are four points. The projection of line segment AB on line CD is

Answer: (8)

Solution:

$\overrightarrow{AB} = \hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $= \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$