Q.9 $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ 2018 AHSEC

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ —— (1)

$\left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right.$ property

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/6+\pi/3-x)}}{\sqrt{\cos(\pi/6+\pi/3-x)}+\sqrt{\sin(\pi/6+\pi/3-x)}}\,dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ —— (ii)

(1) + (ii) $\Rightarrow 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$

$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} \frac{\cancel{(\sqrt{\cos x}+\sqrt{\sin x})}}{\cancel{(\sqrt{\sin x}+\sqrt{\cos x})}}\,dx$

$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} dx$

$\Rightarrow I = \frac{1}{2}[x]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

$\Rightarrow I = \frac{1}{2}\times\frac{\pi}{6}$

$\Rightarrow \boxed{I = \frac{\pi}{12}}$

Q4 $I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$; $x \neq 0$ —— (i)

$$= \int_{-1}^{1} \log\left[\frac{2-(-1+1-x)}{2+(-1+1-x)}\right] dx \qquad \left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right] \text{ Property.}$$

$$I = \int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right) dx \quad \text{—— (ii)}$$

$$\text{(i)} + \text{(ii)} \Rightarrow 2I = \int_{-1}^{1} \left[\log\left(\frac{2-x}{2+x}\right) + \log\left(\frac{2+x}{2-x}\right)\right] dx$$

$$\Rightarrow 2I = \int_{-1}^{1} \log\left[\frac{\cancel{2-x}}{\cancel{2+x}} \times \frac{\cancel{2+x}}{\cancel{2-x}}\right] dx$$

$$\Rightarrow 2I = \int_{-1}^{1} \log 1 \, dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

Q. 6. $\int_1^4 [|x-1|+|x-2|+|x-3|]\,dx$

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$= \int_1^4 |x-1|\,dx + \int_1^4 |x-2|\,dx + \int_1^4 |x-3|\,dx$

$I = I_1 + I_2 + I_3$ [let]

$I_1 = \int_1^4 |x-1|\,dx$

$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$

$= \int_1^4 (x-1)\,dx$

$= \left[\frac{x^2}{2} - x\right]_1^4$

$= \left(\frac{16}{2} - 4\right) - \left(\frac{1}{2} - 1\right) = 4 + \frac{1}{2} = \frac{9}{2}$

$I_2 = \int_1^4 |x-2|\,dx$

$|x-2| = \begin{cases} x-2 & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$

$= \int_1^2 -(x-2)\,dx + \int_2^4 (x-2)\,dx$

Property $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$= -\left[\frac{x^2}{2} - 2x\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$

$= -\left[2 - 4 - \frac{1}{2} + 2\right] + \left[\cancel{8} - \cancel{8} - 2 + 4\right]$

$= 0 + \frac{1}{2} + 2 = \frac{5}{2}$

$I_3 = \int_1^4 |x-3|\,dx$

$= \int_1^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx$

$= -\left[\frac{x^2}{2} - 3x\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$

$= -\left[\frac{9}{2} - 9 - \frac{1}{2} + 3\right] + \left[8 - 12 - \frac{9}{2} + 9\right]$

$= -\frac{8}{2} + 6 + 5 - \frac{9}{2} = -\frac{17}{2} + 11 = \frac{5}{2} = \frac{5}{2}$

$\therefore I = I_1 + I_2 + I_3$

$= \frac{9}{2} + \frac{5}{2} + \frac{5}{2}$

$= \frac{9+5+5}{2}$

$= \frac{19}{2}$ //

Q.3 $\displaystyle\int \frac{\{xf'(x)+1\}}{x\{f(x)+\log x\}}\,dx$

let $f(x) + \log x = t$

$\Rightarrow f'(x) + \frac{1}{x} = dt/dx$ [Differentiating both side w.r.t x.]

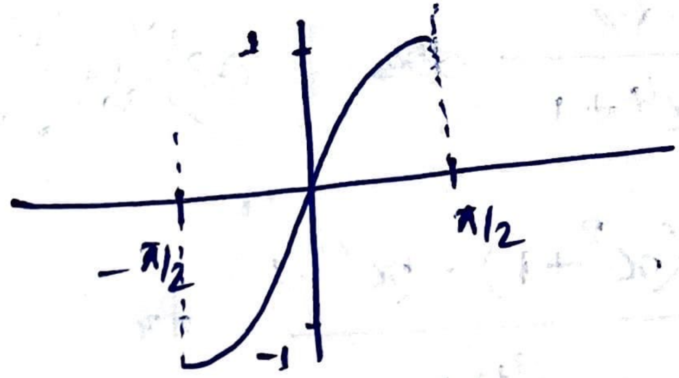
$\Rightarrow \dfrac{xf'(x)+1}{x}\,dx = dt$

$$\int \frac{dt}{t}$$

$= \log t + c$

$= \log|f(x) + \log x| + c$

Q.2 $\displaystyle\int_{-\pi/2}^{\pi/2} |\sin x|\, dx$



$\begin{cases} -\sin x & x < \\ \sin x & x \geq \end{cases}$

$= \displaystyle\int_{-\pi/2}^{0} -\sin x\, dx + \int_{0}^{\pi/2} \sin x\, dx$

$= -\left[-\cos x\right]_{-\pi/2}^{0} + \left[-\cos x\right]_{0}^{\pi/2}$

$= \cos 0 - \cos(-\pi/2) + \left[-\left(\cos \pi/2 - \cos 0\right)\right]$

$= (1 - 0) - (0 - 1)$

$= 1 + 1$

$= 2$

8Q. $I = \int (\sin^{-1}x)^2 \, dx$ 2013 AHSEC

$= \int 1.(\sin^{-1}x)^2 \, dx$

Using By parts.

$= (\sin^{-1}x)^2 \int dx - \int \left[\frac{d}{dx}(\sin^{-1}x)^2 \int dx\right] dx$

$I = x(\sin^{-1}x)^2 - \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \, x \, dx$

$\Rightarrow I = x(\sin^{-1}x)^2 - 2\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}} \, dx$ —— (1)

Now,

$\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}} \, dx$

$\cos t = \sqrt{1-\sin^2 t}$

$= \sqrt{1-x^2}$

Let $\sin^{-1}x = t \Rightarrow x = \sin t$

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

$\int t \sin t \, dt$

$= t\int \sin t \, dt - \int \left[\frac{d}{dt}(t)\int \sin t \, dt\right] dt$

$= -t\cos t + \int \cos t \, dt$

$= -t\cos t + \sin t + c_1$

$= -(\sin^{-1}x)\sqrt{1-x^2} + x + c_1$

Now, (1) $\Rightarrow I = x(\sin^{-1}x)^2 - 2\left[-(\sin^{-1}x)\sqrt{1-x^2} + x + c_1\right]$

$I = x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - 2x + C$

Q. 7.

$$I = \int \frac{1}{x(x^4+1)}\,dx$$ 2013 AHSEC

multiplying numerator and denominator by x^3

$$I = \int \frac{x^3}{x^4(x^4+1)}\,dx$$

let $x^4 = t$

$\Rightarrow 3x^3 dx = dt$

$$I = \frac{1}{3}\int \frac{dt}{t(t+1)}$$

let [using partial fraction]

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$$\Rightarrow \frac{1}{\cancel{t(t+1)}} = \frac{A(t+1)+Bt}{\cancel{t(t+1)}}$$

$\Rightarrow 1 = At + A + Bt$

By comparing the co-eff.

$A+B=0$ $\boxed{A=1}$

$\Rightarrow 1+B=0$

$\Rightarrow \boxed{B=-1}$

$$\therefore I = \frac{1}{3}\int\left(\frac{1}{t} + \frac{-1}{t+1}\right)dt$$

$$= \frac{1}{3}\int\frac{dt}{t} - \frac{1}{3}\int\frac{dt}{t+1}$$

$$= \frac{1}{3}\log t - \frac{1}{3}\log(t+1) + c$$

putting $x^4 = t$

$$\frac{1}{3}\log x^4 - \frac{1}{3}\log(x^4+1) + c$$

$$= \frac{1}{3}\log\frac{x^4}{x^4+1} + c$$

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Integrate : Q.1 $\displaystyle\int \frac{dx}{x^4+1}$ 2024 Nagaon pre board //

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$$= \frac{1}{2}\int \frac{(x^2+1)-(x^2-1)}{x^4+1}\,dx$$

$$= \frac{1}{2}\int \frac{x^2+1}{x^4+1}\,dx - \frac{1}{2}\int \frac{x^2-1}{x^4+1}\,dx$$

$$= \frac{1}{2}\int \frac{\frac{x^2+1}{x^2}}{\frac{x^4+1}{x^2}}\,dx - \frac{1}{2}\int \frac{\frac{x^2-1}{x^2}}{\frac{x^4+1}{x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2.x.\frac{1}{x}}\,dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2.x.\frac{1}{x}}\,dx$$

$$\left[\begin{array}{l}(a-b)^2+2ab = a^2+b^2 \\ (a+b)^2-2ab = a^2+b^2\end{array}\right]$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+(\sqrt{2})^2}\,dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-(\sqrt{2})^2}\,dx$$

let $x-\frac{1}{x} = u$ $\quad\quad$ $x+\frac{1}{x} = v$

$\Rightarrow \left(1+\frac{1}{x^2}\right)dx = du$ $\quad\quad$ $\Rightarrow \left(1-\frac{1}{x^2}\right)dx = dv$

$$= \frac{1}{2}\int \frac{du}{u^2+(\sqrt{2})^2} - \frac{1}{2}\int \frac{dv}{v^2-(\sqrt{2})^2}$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} - \frac{1}{2}\,\frac{1}{2\sqrt{2}}\log\frac{v-\sqrt{2}}{v+\sqrt{2}} + c$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{x-\frac{1}{x}}{\sqrt{2}} - \frac{1}{4\sqrt{2}}\log\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}} + c$$

Q.11. $I = \int \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$ 2024 AHSEC

$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$$ [Using partial fraction]

$$\Rightarrow \frac{x^2+x+1}{\cancel{(x+1)^2(x+1)}} = \frac{A(x+1)(x+2) + B(x+2) + C(x+1)^2}{\cancel{(x+1)^2(x+2)}}$$

$$\Rightarrow x^2+x+1 = A(x^2+3x+2) + Bx + 2B + C(x^2+2x+1)$$

$$\Rightarrow x^2+x+1 = Ax^2+3Ax+2A+Bx+2B+Cx^2+2Cx+C$$

By comparing the co-eff.

$A+C=1$ $\quad$ $3A+B+2C=1$ $\quad$ $2A+2B+C=1$

$\Rightarrow A=1-C$ $\quad$ $\Rightarrow 3A+B+2(1-A)=1$ $\quad$ $2A+2B+(1-A)=1$

$\cancel{\Rightarrow \boxed{C=-1}}$ $\quad$ $\cancel{\Rightarrow A+B=1 \text{ --- (i)}}$ $\quad$ $\cancel{\Rightarrow A+2B=1 \text{ --- (ii)}}$

$\cancel{\text{(ii)}-\text{(i)} \Rightarrow \boxed{B=0}}$

$\cancel{\text{(i)} \Rightarrow A+0=1}$

$\cancel{\Rightarrow \boxed{A=1}}$

$\Rightarrow 3A+B+2-2A=1$ $\quad$ $2A+2B+1-A=1$

$C=1-(-2)$ $\quad$ $\Rightarrow A+B=-1$ --- (i) $\quad$ $\Rightarrow A+2B=0$ --- (ii)

$\boxed{C=3}$

(ii) $-$ (i) $\Rightarrow \boxed{B=1}$

(i) $\Rightarrow A+1=-1$

$\Rightarrow \boxed{A=-2}$

Now $$I = \int\left[\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}\right]dx$$

$$= -2\int\frac{dx}{x+1} + \int\frac{dx}{(x+1)^2} + 3\int\frac{dx}{x+2}$$

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$$= -2\log|x+1| + \frac{(x+1)^{-2+1}}{-2+1} + 3\log|x+2| + c$$

$$= -\log(x+1)^2 - \frac{1}{x+1} + \log(x+2)^3 + c$$

$$= \log\frac{(x+2)^3}{(x+1)^2} - \frac{1}{x+1} + c$$

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