Department of Mathematics

By Rahul Bora

Examination-20.....

Subject:.....

Name:.....

Class:..... Roll No:.....

Stream:..... Section:.....

Date:.....

Invigilator Signature

1 mark Questions:

Important Questions

1. Find the value of $\cos^{-1}\left(\cos\frac{9\pi}{8}\right)$

Ans: $\cos^{-1}\cos\left(\pi+\frac{\pi}{8}\right)$

$= \cos^{-1}\left(-\cos\frac{\pi}{8}\right)$

$= \cos^{-1}\cos\left(\pi-\frac{\pi}{8}\right)$

$= \cos^{-1}\cos\frac{7\pi}{8}$

$= \frac{7\pi}{8}$; $\frac{7\pi}{8} \in [0,\pi]$

2. If A is a row matrix as well as column matrix then what is the order of A.

Ans: 1×1 Eg $A=[5]_{1\times 1}$

3. What is the value of the determinant ?

Ans:
$$\begin{vmatrix} 11 & 12 & 13 \\ 97 & 98 & 99 \\ 22y & 24y & 26y \end{vmatrix} = 2y \begin{vmatrix} 11 & 12 & 13 \\ 97 & 98 & 99 \\ 11 & 12 & 13 \end{vmatrix} = 0$$

(Two rows are identical)

4. $f(x) = \sin x^0$ find $f'(x)$

Ans: $f'(x) = \cos x^0 \frac{dx^0}{dx}$

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5. If A is a square matrix of order 2. Find the value of $|A| + |-A|$

Ans: $|A| + |-A|$

$= |A| + (-1)^2 |A|$

$= |A| + |A|$

$= 2|A|$

6. Are all unit vectors equal.

Ans: No. Not all unit vector equal. because their direction can be different.

7. Find the direction cosine of a line equally inclined to the three co-ordinate axes.

Ans: $\alpha = \beta = \gamma$

$\cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow l = m = n$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow 3l^2 = 1$

$\Rightarrow l^2 = \frac{1}{3}$

$\Rightarrow l = \pm \frac{1}{\sqrt{3}}$

direction cosines are $l = m = n = \pm \frac{1}{\sqrt{3}}$

8. Given that $R = \{(a,b) \mid 3 \text{ divides } a-b\}$ is an equivalence relation in the set of integers Z. What is the numbers of partitions of Z

Ans: 3

9. If A is a square matrix of order 3 such that $|adjA| = 36$ then what is the value of $|A^T|$?

Soln: $|adjA| = 36$

we know.

$|adjA| = |A|^{n-1}$

$\Rightarrow 36 = |A|^{3-1}$

$\Rightarrow |A|^2 = 36$

$\Rightarrow |A| = 6$

Again $|A^T| = |A|$

$\Rightarrow |A^T| = 6.$

(10). $A = [5+i]$, what is the value of $|A|$

Ans: A is the matrix of order 1×1

$|A| = 5+i$

(11) What is the value of $\frac{d}{dx}[x]$, if $x \in (6,7)$. where $[x]$ denotes the greatest integer $\leq x$

Ans: If $x \in (6,7)$

The value of $[x]$ is 6

$\therefore \frac{d}{dx}[x]$

$= \frac{d}{dx}(6)$

$= 0$

12. What is the projection vector of $\vec{a}$ along $\vec{b}$.

Ans: projection of $\vec{a}$ along $\vec{b}$ $= \vec{a} \cdot \hat{b}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

13. Can a vector have angles as 45°, 60°, 120°.

$l = \cos 45° = \frac{1}{\sqrt{2}}$

$m = \cos 60° = \frac{1}{2}$

$n = \cos 120° = \cos(90 + 30°) =$ [redacted]

$= -\sin 30°$

$= -\frac{1}{2}$

$$l^2 + m^2 + n^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2+1+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

Yes vector can have angles 45°, 60° and 120°

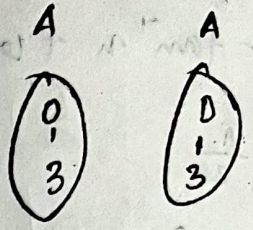
14. If $A=\{0,1,3\}$ what is the number of relations on A.

Ans: $P: A \to A$

$n(A) = 3$

$\therefore$ No of relations $= 2^{3\times 3}$

$= 2^9 = 512.$



15. Find the principal value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

Ans: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

$= \sin^{-1}\sin\left(\pi - \frac{3\pi}{5}\right)$

$= \sin^{-1}\sin\frac{2\pi}{5}$

$= \frac{2\pi}{5}$; $\frac{2\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

16. If $[5,6,7]\,A = [13\ 23]$ what is the order of matrix A.

$\therefore [5\ 6\ 7]_{1\times 3}\ A_{3\times 2}\ [13\ 23]_{1\times 2}$

order of A is 3×2.

17. If matrix A is non-singular and satisfies $A^2 - A + I = 0$, then the inverse of A is equal to

$A^2 - A + I = 0$

$\Rightarrow I = A - A^2$

$IA^{-1} = AA^{-1} - A^2A^{-1}$

$\Rightarrow A^{-1} = I - A \cdot AA^{-1} = I - A$

18. Is the derivative of an even function even?

No, the derivative of an even function may not be even.

For example

$f(x) = x^2$ [even]

Even function means $f(-x) = f(x)$

$f'(x) = 2x$ [odd].

19. Find the value of x such that

$$\cos(\sin^{-1} x) = \frac{1}{2}$$

$$\Rightarrow \sin^{-1} x = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow x = \sin\frac{\pi}{3}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

20. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$. Write down the general values of α.

$$A + A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = 1/2$$

$$\Rightarrow \cos\alpha = \cos\frac{\pi}{3}$$

$$\therefore \alpha = 2n\pi \pm \frac{\pi}{3}; \quad n \in Z$$

21. If A is a matrix of order 3×4 and B is a matrix of order 4×5 what is the order of matrix $(AB)^T$

Ans: order of AB →

$$A_{3\times4}\, B_{4\times5} = (AB)_{3\times5}$$

$\therefore (AB)^T$; order $= 5 \times 3$.

22. Let A be a 3×3 determinant and $|A| = 7$ Find the value of $|2A|$

Ans:
$$|2A| = 2^3 |A|$$
$$= 8 \times 7$$
$$= 56.$$

23. Write down the range of $\operatorname{cosec}^{-1} x$

Ans: Range of $\operatorname{cosec}^{-1} x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

24. Find the degree and order of DE.

$$xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

Ans: order = 2
degree = 1

(25) If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ are such that $\vec{a} \perp \vec{b}$, what is the value of λ.

Ans: $\because \vec{a} \perp \vec{b}$.

$\therefore \vec{a} \cdot \vec{b} = 0$

$\Rightarrow (2\hat{i} + 4\hat{j} - \hat{k}) \cdot (3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

$\Rightarrow -2 - \lambda = 0$

$\Rightarrow \boxed{\lambda = -2}$

(26) What is the vector equation of the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$

Ans: $\vec{a} = -\hat{i} + 2\hat{k}$

$\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

vector equation $\Rightarrow \vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\Rightarrow \vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(27) Let $A = \{x : 1 < x < 10, x \text{ is an odd natural no}\}$

$B = \{y : 90 < y < 100, y \text{ is a prime number}\}$

Write the number of relation from A and

Ans: $A = \{3, 5, 7, 9\}$ $B = \{97\}$

$n(A) = 4$ $n(B) = 1$

No of relation $= 2^{4 \times 1} = 2^4 = 16$

(28) find all the positive values of 2×2 determinants whose entries are from the set $\{-1,0,1\}$

Ans:

$$\begin{bmatrix} 3\text{ways} & 3\text{ways} \\ 3\text{ways} & 3\text{ways} \end{bmatrix}$$

Using Fundamental principle of counting,

Total no of 2×2 determinants $= 3\times3\times3\times3$

$= 3^4 = 81$

(29) Let A be a skew symmetric matrix of odd order. Write the values of $|A|$

Ans: $A^T = -A$ [As A is skew symmetric]

$|A^T| = |-A|$

$\Rightarrow |A| = (-1)^n |A|$

$\Rightarrow |A| - (-1)^n |A| = 0$

$\Rightarrow |A|\left[1-(-1)^n\right] = 0$

As n is odd, $1-(-1)^n \neq 0$

$|A| = 0$

(30) If $\cos^{-1}x - \sin^{-1}x = 0$, write down the value of x.

Ans: $\cos^{-1}x - \sin^{-1}x = 0$

$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1}x\right) - \sin^{-1}x = 0$

$\Rightarrow \frac{\pi}{2} - 2\sin^{-1}x = 0$

$\Rightarrow 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{4}$

$\Rightarrow x = \sin\frac{\pi}{4}$

$\Rightarrow \boxed{x = \frac{1}{\sqrt{2}}}$

(31) If A be a square matrix of order 2 whose determinant is $-i$, Find the value of $|A(adjA)|$

Ans: $|A(adjA)| = |A|^n$

$\Rightarrow |A(adjA)| = (-i)^2 = i^2 = -1$

(32) What is the number of all possible 2×3 matrices with entries 0 or 1.?

Ans: order $= 2\times3$

$$\begin{bmatrix} 2\text{ways} & 2\text{ways} & 2\text{ways} \\ 2\text{ways} & 2\text{ways} & 2\text{ways} \end{bmatrix}$$

$\therefore$ Total number of matrices $= 2\times2\times2\times2\times2\times2$

$= 2^6 = 64.$

(33) What is the value of $\lim f(x)$ if a function f is continuous at $x=0$ and $f(0)=7$

Ans: $\lim f(x) = f(0) = 7$

(34) State whether it is true or false: "The derivative of an even function is always even function"

Ans: No it is false [see Q. 18]

(35) If $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors. What is the value of

$(2\hat{a}+3\hat{b})\cdot(4\hat{a}-5\hat{b})$

Ans: to Q35

$$(2\hat{a}+3\hat{b}).(4\hat{a}-5\hat{b})$$

$$= 2\hat{a}.4\hat{a} - 2\hat{a}.5\hat{b} + 3\hat{b}.4\hat{a} - 3\hat{b}.5\hat{b}$$

$$= 8|\hat{a}|^2 - 10\,\hat{a}.\hat{b} + 3\hat{b}.\hat{a} - 15|\hat{b}|^2$$

$$= 8 - 0 + 0 - 15$$

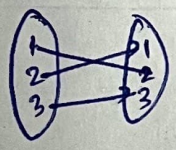
$$= -7$$

36. For a given set $A=\{1,2,3\}$. state whether the following statement is true or false. Justify your answer.

"An onto function from A to A is always one-one"

Ans:

$f: A \to A$



Onto function means for any $y \in$ codomain. there is some preimage in domain. for the given set $A=\{1,2,3\}$ if it is onto it must be one-one.

So the statement is true.

37. Differentiate w.r.t x $\log(\log(\log x^n))$, $x>0$ and n is constant.

Ans: $y = \log(\log(\log x^n))$

Differentiating w.r.t x.

$$\frac{dy}{dx} = \frac{1}{\log(\log x^n)} \frac{d}{dx} \log(\log x^n)$$

$$= \frac{1}{\log(\log x^n)} \frac{1}{\log x^n} \frac{d}{dx}(\log x^n)$$

$$= \frac{1}{\log(\log x^n)} \frac{1}{\log x^n} \frac{1}{x^n} \frac{d}{dx}(x^n)$$

$$= \frac{1}{\log(\log x^n)} \frac{1}{\log x^n} \frac{1}{x^n} n x^{n-1}$$

38. What is the maximum value of the function $\sin x + \sqrt{3}\cos x$

Ans:

$$f(x) = \sin x + \sqrt{3}\cos x$$

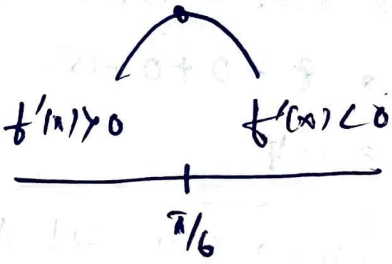
$$f'(x) = \cos x - \sqrt{3}\sin x = 0$$

$$\Rightarrow \sqrt{3}\sin x = \cos x$$

$$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6}. \text{ [critical point]}$$



$x = n\pi + \frac{\pi}{6}$

↑

maximum values = ?

$x = \frac{\pi}{6}$ point of local maxima.

∴ Maximum value

$$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} + \sqrt{3}\cos\frac{\pi}{6}$$

$$= \frac{1}{2} + \sqrt{3}\,\frac{\sqrt{3}}{2}$$

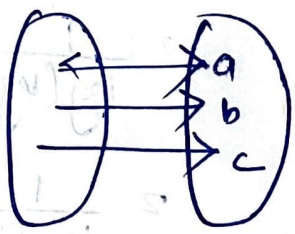
$$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \;//$$

39. Let A and B be any two given set. If $f: A \to B$ is onto function, then find the range of f

Ans: For onto function, Every $y \in$ co-domain, there is some preimage in domain.

i.e every element of codomain (B) belongs to Range.

Range = B



40. What are the direction cosine of x axis.

Ans: x axis makes $0°, 90°, 90°$ with x, y and z axis's

So Direction cosines are

$$l = \cos 0 \quad m = \cos 90 \quad n = \cos 90$$

$$l = 1 \quad m = 0 \quad n = 0$$

DC's are $(1, 0, 0)$

41. Let A be a matrix of order 3×3 such that $|A| = 9$. Find the value of $|-3A^{-1}|$

Ans:

$$|-3A^{-1}|$$

$$= (-3)^3 |A^{-1}|$$

$$= -27 \frac{1}{|A|}$$

$$= \frac{-27}{9}$$

$$= -3$$

42. Differentiate $\sin x$ w.r.t e^x

Ans: $\frac{d(\sin x)}{d(e^x)} = ?$

Let $U = \sin x \quad V = e^x$

$\Rightarrow \frac{du}{dx} = \cos x \quad \frac{dv}{dx} = e^x$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\cos x}{e^x} \quad //$$