RAMANUJAN SENIOR SECONDARY SCHOOL

PRE FINAL 1 EXAMINATION 2023

SUB: MATHEMATICS

YEAR: H.S 2^{ND} YEAR

TIME: 3 HOURS FULL MARKS: 100

1. Answer the following questions. $1 \times 10 = 10$

(i) For a given set $A = \{1,2,3\}$ .Find the number of all onto functions from A to A

(ii) For a given set $A = \{1,2,3\}$. State whether the following statement is true or false. Justify your answer.

"An onto function from A to A is always one – one."

(iii) Find the principal value of $cos^{-1}\left(cos\frac{13\pi}{6}\right)$

(iv) If A is square matrix of order 2 whose determinant is $-i$, find the value of $|A(adjA)|$

(v) Differentiate with respect to x $\log\{log(logx^n)\}, x > 0$ and n is a constant.

(vi) What is the maximum value of the function $sinx + \sqrt{3}cosx$

(vii) Evaluate $\int_{-c}^{c}[f(a-x) - f(a+x)]dx$

(viii) Write the geometrical interpretation of $\int_a^b f(x)dx$.

(ix) Find the number of arbitrary constants in the particular solution of a differential equation of third order.

(x) Find the distance if the point (2,3,4) from the x- axis.

0 . . . 12

2. Show that the relation R in the set, $A = \{x \in Z: 0 \leq x \leq 12\}$ given by

$R = \{(a,b): |a-b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1. 3 +1 = 4

3. Define injective function. Check the injectivity of the following functions 1+ 3 = 4

(a) $f: N \to N$ given by $f(x) = x^2$ (ii) $f: Z \to Z$ given by $f(x) = x^2$

4. Show that $sin^{-1}\frac{12}{13} + cos^{-1}\frac{4}{5} + tan^{-1}\frac{63}{16} = \pi$ 4

OR

Show that $tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x, x \in \left[-\frac{1}{\sqrt{2}}, 1\right]$

5. Find the matrix X , if $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$ 4

Or

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below

Markets	Products		
I	10000	2000	18000
II	6000	20000	8000

If unit sale prices of x, y and z are $Rs\ 2.50$, $Rs\ 1.50$ and $Rs\ 1.00$ respectively, Find the total revenue in each market with the help of matrix algebra. 4

6. Find the points of discontinuity of f, where 4

$$f(x)=\begin{cases}\frac{\sin x}{x}, & if\ x<0\\ x+1, & if\ x\geq 0\end{cases}$$

7. Find the derivative of the function given by $f(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$. 4

OR

Find $\frac{dy}{dx}$ if $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ 4

8. If $y=(\tan^{-1}x)^2$, Show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$ 4

OR

Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one –sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4cm$? 4

9. Integrate the followings : i) $\int\frac{\tan^4\sqrt{x}*\sec^2\sqrt{x}}{\sqrt{x}}dx$ (ii) $\int\frac{\{xf'(x)+1\}}{x\{f(x)+\log x\}}dx$ 2 + 2 = 4

OR

$$\int\frac{x+2}{2x^2+6x+5}dx$$ 4

10. Evaluate $\int_0^{\frac{\pi}{2}}\log\sin x\,dx$ 4

OR

Prove that $\int_{-a}^{a}f(x)dx=0$, when f is an odd function. Hence evaluate $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$. 2 + 2 = 4

11. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ 4

OR

Find the distance between the lines l_1 and l_2 given by

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ 4

12. Find the angle between the pair of lines 4

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} \text{ and } \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

Or

The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also find its area. $[\vec{a} \times \vec{b}]$ 4

13. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is

$\frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$ 4

OR

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the co-ordinates of the point. 4

14. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. 6

OR

$A' = \frac{1}{|A|} \text{adj } A$

Solve the system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ 6

15. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, Prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b. 6

OR

Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by

$(x + y + 1) = A(1 - x - y - 2xy)$, where A is parameter. 6

16. (i) Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$ $3+3=6$

(ii) Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is strictly increasing.

OR

(i) Show that the semi vertical angle of the cone of the maximum volume and of given slant height is $tan^{-1}\sqrt{2}$.

(ii) Find the maximum profit that a company can make, if the profit function is given by

$P(x) = 41 - 24x - 18x^2$ $4+2=6$

17. Find the area of the region bounded by the line $y = 3x + 2$, the $x - axis$ and the ordinates

$x = -1$ and $x = 1$. 6

OR

Find the area bounded by the curve $y = x|x|$, x $axis$ and the lines $x = -1$ and $x = 1$. 6

18. (i) Prove that $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$, if $\vec{a}, \vec{b}$ are perpendicular,

given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$.

(ii) If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vectors

$\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}, and\ \vec{c}$. $3+3=6$

19. Solve the Linear Programming Problem graphically 6

Maximise $Z = 5x + 3y$

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10, x, y \geq 0$

OR

Solve the Linear Programming Problem graphically 6

Minimise and Maximise $Z = 3x + 9y$

Subject to $x + 3y \leq 60$, $x + y \geq 10, x \leq y\, x, y \geq 0$

20. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly. 6

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