MOST IMPORTANT QUESTIONS FOR AHSEC-2024 MATHEMATICS (LECTURE 11)

Relations and functions:

Q.1. Determine whether the following relation is equivalence or not!

R in the set $A=\{1,2,3,\ldots,13,14\}$ defined as

$$R=\{(x,y): 3x-y=0\}$$

$A=\{1,2,3\}$

$R: A \to A$

Reflexive Relation

for all $a \in A$

$(a,a) \in R$

$R=\{(1,1),(2,2),(3,3),(3,1),(1,2)\}$

Transitive Relation

for all $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

$R=\{(1,2),(2,1),(1,1),(2,2),(1,3),(3,1),(2,3),(3,2)\}$

Symmetric Relation

for all $(a,b) \in R$

$\Rightarrow (b,a) \in R$

$R=\{(1,2),(2,1),(3,1)\}$

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Q.1. Determine whether the following relation is equivalence or not?

R in the set $A=\{1,2,3,\ldots,13,14\}$ defined as

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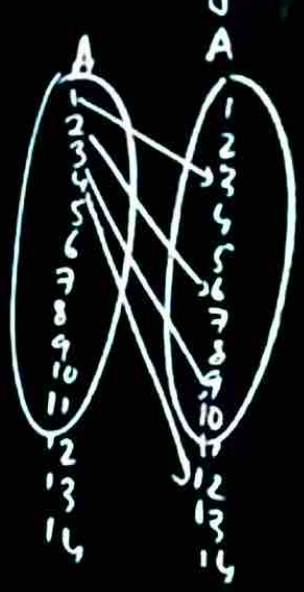
Sol^{n} Given $R: A \to A$, $A=\{1,2,3,\ldots,13,14\}$

$$R=\{(x,y): 3x-y=0\}$$

$$=\{(1,3),(2,6),(3,9),(4,12)\}$$



Reflexive: For $3 \in A$, $(3,3) \notin R$

$\therefore$ It is not reflexive

Symmetric For $(1,3) \in R \Rightarrow (3,1) \notin R$

It is not symmetric

Transitive for $(1,3) \in R$ and $(3,9) \in R$

$\Rightarrow (1,9) \notin R$. it is not transitive.

MOST IMPORTANT QUESTIONS FOR AHSEC-2024 MATHEMATICS (LECTURE 11)

Relations and functions:

Q. 2. Show that the relation R in the set Z of integers given by $R = \{(a,b) : 2 \text{ divides } a-b\}$ is an equivalene relation.

Soln Given $R : Z \to Z$, $R = \{(a,b) : 2 \text{ divides } a-b\}$

Reflexive: for $a \in Z$

$a - a = 0 = 2r$

$(a,a) \in R$ ← This is reflexive

$6 = 2 \times 3$

$8 = 2 \times 4$

$30 = 2 \times 15$

Symmetric for $(a,b) \in R$

$\Rightarrow a - b = 2r$

$\Rightarrow b - a = -2r = 2(-r)$ This symmetric

$\Rightarrow (b,a) \in R$ ←

Transitive For $(a,b) \in R$ $(b,c) \in R$

$\Rightarrow a - b = 2r_1$ (I) $\quad b - c = 2r_2$ (II)

(I) + (II) $\Rightarrow a - b + b - c = 2(r_1 + r_2)$

$\Rightarrow a - c = 2(r_1 + r_2)$

$\Rightarrow (a,c) \in R$ //

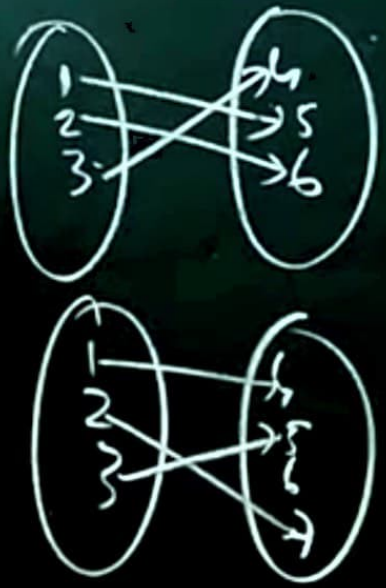
MOST IMPORTANT QUESTIONS FOR AHSEC-2024 MATHEMATICS (LECTURE 11)

Relations and functions

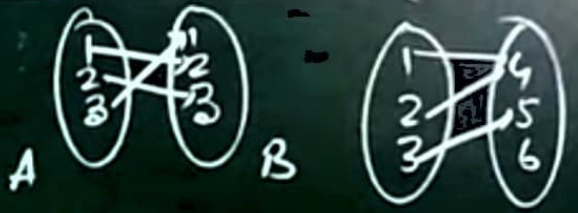
Q. 3. Prove that the function $f: R \to R$ given by $f(x) = 2x$ is one one and onto

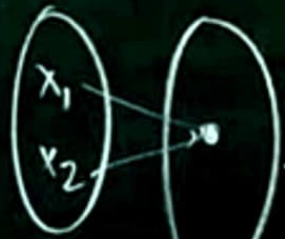
Soln

Onto function / Surjective fn



One one function (Injective fn)





$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

MOST IMPORTANT QUESTIONS FOR AHSEC-2026 MATHEMATICS (LECTURE 11)

Relations and functions:

Q. 3. Prove that the function $f: N \to N$ given by $f(x) = 2x$ is one one and not onto.

Soln Given $f: N \to N$ $f(x) = 2x$

One one let x_1 and $x_2 \in$ domain.

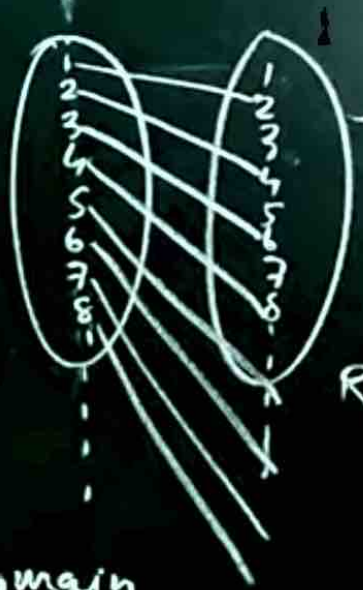
$f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

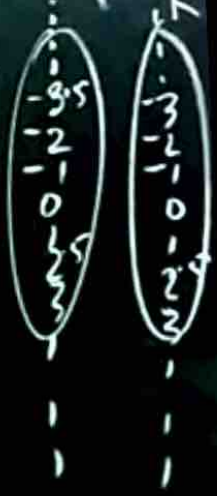
$\Rightarrow x_1 = x_2$ This is one one

Onto For $5 \in$ codomain, there is no preimage in domain

This is not onto.



$f: R \to R$



Inverse trigonometric functions

Q. 1. $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Let $x = \cos 2\theta \Rightarrow 2\theta = \cos^{-1}x \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

L.H.S $= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}\right)$

$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)$

$= \frac{\pi}{4}-\theta$

$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

$=$ R.H.S //

Inverse trigonometric functions

Q.2 Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$

$\sin 2x = 2\sin x \cos x$

$\Rightarrow \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

L.H.S $= \cot^{-1}\left[\frac{\sqrt{1+2\sin x/2\cos x/2}+\sqrt{1-2\sin x/2\cos x/2}}{\sqrt{1+2\sin x/2\cos x/2}-\sqrt{1-2\sin x/2\cos x/2}}\right]$

$= \cot^{-1}\left[\frac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin x/2\cos x/2}+\sqrt{\cos^2 x/2+\sin^2 x/2-2\sin x/2\cos x/2}}{\sqrt{\quad}+\sqrt{\quad}}\right]$

$= \cot^{-1}\left[\frac{\sqrt{(\cos x/2+\sin x/2)^2}+\sqrt{(\cos x/2-\sin x/2)^2}}{\sqrt{(\cos x/2+\sin x/2)^2}-\sqrt{(\cos x/2-\sin x/2)^2}}\right]$

$\cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin x/2}\right]$

$= \cot^{-1}\cot x/2$

$= \frac{x}{2} = R.H.S$

Relations and Functions important Questions

Ex 1.1.

Example 3, 4, 5

Exercise 2, 8, 9(i), 12

Ex 1.2

Example 11

Exercise 3, 4, 9, 10.

Miscellaneous 4, 5, 7, 16, 17, 19

Inverse trigono metric function

Ex 2.1

Example 1

Ex 2.2

Example (4) Express $\tan^{-1}\frac{\cos x}{1-\sin x}$, - - - - -

Exercise (12) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$